Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 263 JANUARY, 1957 No. 1

# A STUDY OF THE FEASIBILITY OF TONAL BRAILLE *

BY

WALLACE E. FRANK [1]

SUMMARY

The program set forth in this paper was undertaken to determine the feasibility of a new auditory system of reading for the blind, called "tonal Braille." Since available data did not appear to provide an adequate basis for predicting the learnability and acceptability of this method of communication, an exploratory test program was initiated. This paper reviews the considerations which led to the program, the data obtained and the conclusions.

## NEED FOR A NEW READING SYSTEM FOR THE BLIND

It is estimated that less than 25 per cent of the legally blind read Braille at all well. The simple 25–75 per cent figures, although probably very conservative, are misleading; they must be considered in the proper perspective. Sixty per cent of the blind population is composed of people past 60 years of age. For most of them, learning new techniques of reading, which involve tactile or auditory sense, is a difficult task. Habits of a lifetime are not readily changed.

The lack of adjustment to blindness is another factor for the small usage of Braille. Certainly it is never easy to admit to oneself that vision is gone, that there is a severe handicap to be overcome, that one is among "the blind." Rejection of all things associated with blindness may persist for many years after sight has been lost. To most, learning Braille is an admission of being blind permanently. Some people simply do not have enough tactile sense to become Braille readers. Some of the legally blind can read conventional printed material.

Infrequently an adult becomes a good Braille reader. This is not to say that many adults do not master reading by touch; but, by and large, the relaxed, rapid readers of Braille are persons who learned the

* Supported by the Philadelphia Branch of the Pennsylvania Association for the Blind, Inc., with funds supplied by the Benjamin Rosenthal Foundation, Inc.

[1] Chief, Bioengineering Section of The Franklin Institute Laboratories, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

system in their early years. Children who attend schools for the blind and learn Braille at an early age frequently acquire great skill in reading. Speeds of reading with good comprehension are often very high. No great fatigue limits their reading time. Embossed printing certainly has great potential for those who master it.

The blind who lose their sight after childhood are about representative of the general population in education and intelligence. Even though we live in a country where literacy is almost universal, most of our people do not have a great interest in reading. Magazines and newspapers sell by the millions, but many readers would not make much of an effort to get them if greater expense or trouble were required. People who read little by sight can hardly be expected to go to great lengths to master Braille, just in order to read the small amount they were in the havit of covering.

The need is for an easily learned technique which will serve not only the reading needs of many of the 75 per cent of the blind who cannot read except through a sighted assistant (including records), but also may be of use in adjusting those who have lost their sight and as means of teaching Braille.

Another important and related aspect of blindness is that loss of sight is usually accompanied by low earning power. The blind cannot afford expensive devices in the solution of their day-to-day problems.

It seemed to us there was a possibility that Braille could be converted into a method of audible reading that might solve some of the problems discussed above. Without a trial, no intelligent decision could be made. We could not even decide what to try except that it should satisfy the following requirements:

*a.* The system must be directly related to Braille. (This is necessary so that no expensive "recognition" circuits would be required. A dot in the Braille code should have a fixed relationship to a sound.)
*b.* The output must be audible.

The device for reading embossed printing should consist only of a head which could be drawn over a page to produce an audible signal that is an exact counterpart of the Braille. We call the new system "tonal Braille."

The insistence upon the close resemblance to Braille also stems from a desire to utilize the Braille material now available and to stimulate greater production of Braille in the future. Tonal Braille could in no sense outmode Braille; it might be an additional means of reading for some of the blind. It would fit in with tactile reading, the talking book, special readers, etc. It might also prove to be useful in teaching Braille.

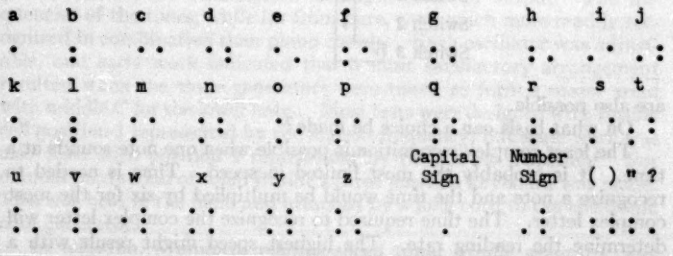
Before discussing tonal Braille let us first consider Braille. All Braille is represented by various combinations of the raised dots in a Braille cell, which consists of six dots, arranged in two columns of three

dots each. Braille is read horizontally, from left to right as is conventional printed material. The entire Braille alphabet is formed by various combinations of from one to six dots. Contracted Braille, Grade 1½, 2, etc., uses the same cell but assigns special meaning to some conventional letters and combinations, and also uses special symbols for other contractions. Part of the explanation of the high speed attained by good Braille readers is that they read, not uncontracted Grade 1, but highly contracted Grade 2.

The dots in the cell are numbered one to three in the first column, from top to bottom, and four to six in the second column.

Braille
• 1 • 4
• 2 • 5
• 3 • 6

*Braille Alphabet and Numerals.*

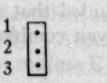



The capital sign, dot 6, placed before a letter, makes it a capital.
The numeral sign, dots 3, 4, 5, 6, placed before a character, makes it a figure and not a letter.

## TONAL BRAILLE

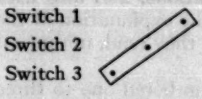
A simple way to convert Braille into a tone system is to represent each dot in a particular horizontal level by a note. To accomplish the conversion from Braille to sound, a switch is connected to each of three mechanical fingers which pass over the Braille. One finger passes over dots 1 and 4, another over dots 2 and 5, and the third over dots 3 and 6. Signals for column two are exactly like those for column one.

If the switches are arranged vertically,



the most complex chord will consist of three notes. Combinations of any two as well as single notes are possible. The possibilities are three single notes, three pairs and a three-note chord.

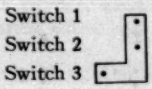
If the switches are arranged at an angle




it may be that only one note will be heard at a time.

Slight angularity may be used so that each level starts its sound at a different time, as the reading head is drawn across the page, but persists while one or two others start.

Special arrangements such as,




are also possible.

On what basis can a choice be made?

The least complex recognition is possible when one note sounds at a time. It is probably the most limited in speed. Time is needed to recognize a note and the time would be multiplied by six for the most complex letter. The time required to recognize the complex letter will determine the reading rate. The highest speed might result with a single chord per column. The chord is not always easy to recognize because of its complexity. Systems of conversion such as one level sounded at a different time from the other two are compromises between recognition time and complexity.

One important characteristic of a reading system is initial learnability, for example: Is progress rapid enough and promising enough to hold the interest of the blind who can not or will not learn Braille? Another very important characteristic is the maximum practical speed of reading to be expected. The design that is best in one respect is not necessarily best in the other.

An experiment was performed to determine a person's ability to recognize the signals when all combinations of three notes were sounded. For this test, about a dozen people were selected at random from personnel at the Laboratories. It was concluded that all of them could be taught to recognize any one of the seven combinations possible with three notes.

We thought that a tone system might permit even higher reading rates than those first indicated. Tone patterns for words and phrases could be sufficiently distinctive to allow direct recognition without reconstruction from the letter content. For such recognition the clues are the chord structure, loudness and rhythm.

### TEACHING DEVICE

In order to evaluate any one of the proposed variations of tonal Braille it was necessary that equipment be designed and constructed which would permit the production of Braille material. Preliminary work was carried out with a piano. Magnetic tape recordings in various systems were made of the alphabet and simple words. The types were tested informally by a small jury. They found that too many overtones were present in the piano notes.

Since a simple device could convert one column of the Braille cell into sound at a time, it was decided to build a simple electronic tone generator with three oscillators feeding a common output. The frequencies of the tones, while far from pure, were much more readily recognized in combination than piano chords. Each oscillator was adjustable, and early work indicated that a most satisfactory arrangement resulted when the three generators were tuned to form a major triad with middle C for the lower note. Most tests were designed with Braille cell position 1 represented by the highest frequency, but some work was also tried with position 1 represented by the lowest frequency. For manual production of tonal Braille a three position keyboard was made. This was held in the hand and attached by a long cord to the electronic note generator.

In addition, a uniform reading-speed tonal Braille generator was made. This device worked from paper tapes prepared on a Braille writer. They were passed at uniform speed under three fingers which controlled the sound.

### TEACHING METHODS

A study of the Braille alphabet will show that the complexity of the letters increases from *a* to *z*. Note that letters A through J do not involve any of the bottom dots. Thus, the letters A to J in tonal Braille involve a maximum of two notes in a chord.

In the earlier lessons these letters are taught, and simple words are composed only with the letters studied. A sample lesson appears in the Appendix. Many magnetic tapes for tonal Braille lessons were made with the uniform rate generator.

The teaching of blind students proceeded very rapidly during the early stages. Few students had any difficulty with the letters A through J. They learned rapidly. Although many mistakes were made and there was a tendency to confuse E and I, D and F, H and J, due to trans-

position, generally speaking, progress was excellent. Even at the end of the first lesson, the student was reading simple words. This experience almost inevitably aroused his enthusiasm. However, difficulties arose when there was a possibility of three notes in the chord. One of the main problems was to distinguish between letters K and L and all other letters requiring a decision as to whether a chord consisted of all three notes or only the upper and lower ones. While experience showed that most of the students could learn this distinction, it represented the first serious slow-down in the learning of tonal Braille.

Following the alphabet exercises, the student proceeded to what were called sentence groups. Here, emphasis was put on the reading of words in a sentence. The instructor normally required the student to state aloud his interpretation of tonal Braille. After the sentence groups, word studies were used. These words were made up largely of the difficult letters, and the student began to recognize words as tone pattern entities, apart from their letter construction. After practice with these sentences having no particular story, a group of poems and a short essay were tried. At any time in a lesson when additional instruction, or review not on the tapes, was necessary, the instructor, using the keyboard, could compose the needed material for the student.

It seemed that learning tonal Braille might well be very similar in many respects to learning tactile Braille. Therefore a Braille primer was secured. Lesson material was prepared with the Braille writer and from these magnetic tapes were made in tonal Braille. The general organization of the lessons in the Braille primer was followed, but extra reviews were added. The Braille tape was run through the converting machine at a letter speed equivalent to 50 words per minute for Grade 1 Braille. This was the speed at which the Braille cells were presented. Naturally, a beginner could not read at this rate, so many spaces were inserted between each cell. This spacing, as a matter of fact, was reduced as the lessons progressed. The idea was that the student should recognize the tonal Braille letters when the time of presentation was short. When a student missed too many letters at this speed, the instructor wrote additional letters by hand, keeping the tones sounding for a longer time. A series of lessons, each approximately one hour long, were prepared and recorded on magnetic tape.

During all of the teaching with the prepared magnetic tapes, the instructor would frequently stop the tape and repeat portions not understood by the student. He might also repeat more slowly or in different context by using the hand keyboard.

With the more advanced students, some attempt was made to give them meaningful messages by using the hand keyboard. These sentences would be about subjects of interest to them and were frequently light in spirit. Not only was the content helpful in keeping attention, but the instructor also adjusted the presentation rate to the difficulty of

the material. Thus, short familiar words were given at a very rapid rate, whereas difficult or unexpected ones were presented slowly enough to permit construction from the letters. The speed of presentation was not even uniform throughout the word since, for instance, an expected or familiar suffix could be read at much higher speed.

In a few instances tape recordings of teaching sessions were made as a guide to us in designing the lessons and analyzing student progress.

### TEACHING ENVIRONMENT

Almost all of the lessons were given to individual students, alone in a room with the instructor. Many of these were given at The Franklin Institute. In a few instances group instruction was tried, with the largest class consisting of four students. Where the pupils were of about equal skill, the group method seemed to be effective. Particularly when the hand keyboard was used with impromptu material, interest was high and learning was rapid.

Late in the program, a record for self-practice was made. Copies were loaned to several students. While they were interested in this method of learning, the breadth of material on the single record was so small that the effect was slight.

Towards the close of the program, one excellent student was provided with the tone generator and a crude scanning head for use with standard Braille. This was to be used for self-teaching. Unfortunately, the scanning head was so unreliable that the experiment was of almost no value.

### STUDENTS

While it was recognized that tonal Braille might find greatest use where Braille was rejected, most of our subjects knew Braille. The object of this program was to determine the learnability of tonal Braille and to estimate the potential reading rates to be expected. Relative acceptability of tonal Braille as compared with Braille is another issue.

Our subjects ranged in age from 22 to 81 years. Some of the students were employed, some were housewives, and others were retired. A number lived in residential institutions and were taught there. All were volunteers. None were paid for their work with tonal Braille. They stayed with it because they were interested or because they felt the study was worth while. They were reimbursed only for travel expenses.

In order to save time in this program, we chose mostly adults who knew the Braille alphabet. Thus, we were spared the time of teaching it. Learning to visualize the Braille alphabet is not a very difficult task, even when starting from scratch. We recognize, of course, that the choice may have been disadvantageous. Perhaps progress in tonal Braille was hampered by a knowledge of Braille, since this required a translation into Braille for comprehension. It is possible that teaching

the tonal Braille alphabet directly would, in the long run, have been a more efficient procedure.

### STUDENT PERFORMANCE

The accompanying table summarizes the results with sixteen students. Other students were started but did not advance as far as those listed. Most of these were not interested in any new reading system. A surprisingly small number were annoyed by the tones.

*Tonal Braille Students.*

| Student | Age | No. of Sessions | Remarks |
|---|---|---|---|
| 1 | 81 | 12 | Hard of hearing but very interested. |
| 2 | 72 | 6 | Learned alphabet—interested. |
| 3 | 45 | 8 | Learned alphabet, poor tone recognition although a singer. |
| 4 | 22 | 20 | Through sentence drills—slow learner but interested. |
| 5 | 70 | 10 | No Braille experience—poor general health prevented more work. |
| 6 | 23 | 11 | Could not distinguish K from L. |
| 7 | 30 | 6 | Learned alphabet—lost interest. |
| 8 | 60 | 9 | Learned alphabet, interested—busy with other activities. |
| 9 | 35 | 14 | Proficient—lost interest. |
| 10 | 40 | 6 | Learned alphabet—lost interest. |
| 11 | 43 | 49 | Interested throughout—no letter recognition problem. |
| 12 | 30 | 42 | Interested throughout—no letter recognition problem. At 40 sessions 23 wpm. |
| 13* | 60 | 44 | Same as 12 |
| 14* | 60 | 45 | Same as 12 |
| 15* | 60 | 25 | Fair |
| 16 | 26 | 50 | Interested throughout—23 wpm. (Used generator on his own—pickup head for reading from Braille directly was unsatisfactory.) |

* Instructed as a group.

It should be noted that almost all of the time spent with tonal Braille, except in the case noted below, was spent with the instructor. There was almost no self-teaching or practice alone. One 12-in. record was made and loaned to several students, but its effect upon learning, while definitely favorable, was small. One student did make a serious effort to use a Braille pick-up head to transform Braille to tonal Braille, but mechanical difficulties prevented much success.

One of the most important observations made was that students generally could tolerate extended sessions of listening to tonal Braille. Many did not want to terminate at the end of one hour, and, on several occasions, sessions lasted two hours. Another important observation was that, at the end of the first session, the subject could read simple words by tonal Braille. Progress was continuous through the letter J. Only when K and L appeared was there a serious slowdown in the mastering of the system.

Students learn to recognize letters given at the rate of 50 words per minute, but widely spaced, in the first lesson. (This is equal to a much higher rate in Grade 2 Braille.) The duration of the signal is the same as it is when reading Grade 1 at 50 words per minute. This duration is long enough. The tones do not merge at this rate. The fact that more time is taken to remember reflects a lack of practice and familiarity.

We were disappointed with the maximum reading speed that was attained. While 25 words per minute may represent an important achievement for a blind person who can not, and perhaps will not, read Braille, it is very slow reading.

During the program, several short tapes were made at equivalent Grade 1 speed of 50 words per minute. An individual who was unfamiliar with tonal Braille could easily recognize the pattern of a sentence after listening to it for a short time. We could have started out, as many do, with modern elementary courses in reading, by teaching words, not letters. If we had done so, we would undoubtedly have had several students develop a very high reading speed with a limited vocabulary.

Since tonal Braille was directed, in part, toward the problems of the newly blinded adult, it was felt that there should be no reduction in vocabulary in order to broaden the horizons of reading as rapidly as possible. Once the student mastered the alphabet, he would be ready to read any Grade 1 material and when he knew the rules of contraction, contracted Braille.

None of our students had enough practice to pass from letter- to word-reading. How much practice is required is still not known. Of course, simple recurring words were recognized rapidly, as were familiar suffixes and more complex unusual words, which appeared several times in context. But, in any paragraph, only a negligible portion of the material was recognized directly; most of it had to be reconstructed letter by letter.

## CONCLUSIONS

Tonal Braille, as we have tried it, can be learned. It is not tiring. Rather, it has fascinated our students. With the practice that our students had, the reading speed reached by them was low. None of the students reached a state of proficiency in which they read words as a whole.

Other experiments showed word reading to be possible. Letters were given at a reasonably high rate. Self-teaching methods must be provided if the student is to make adequate progress.

No system of tonal Braille except that which reads an entire column of the Braille cell at a time was tried. Since the introduction of the third note raises a serious problem, some "staggered" system should be tried.

No effort was made to determine the usefulness of tonal Braille in teaching Braille.

Further work is needed to establish the feasibility of tonal Braille. It should include:

*a.* Preparation of self-teaching material (can be used with students already trained).
*b.* Investigation of tonal Braille without the K-L problem.
*c.* Use of tonal Braille in connection with teaching tactile Braille.

*Acknowledgments*

The idea of tonal Braille first arose during a conference between Dr. George J. Kreier, Jr. and the author. Dr. Kreier also gave valuable advice during the course of the program. Mr. William McDonald contributed not only to the preparation of the lessons and served as instructor for all students, but also did much of the work of guiding and evaluating the work all during the program.

## APPENDIX

*Sample Lesson*

This sample lesson, edited slightly for publication purposes, is presented as transcribed from a tape recording that was made during the course of instruction.

### EXERCISE NO. 1.*

Group 1

The first word: ace
Second word: bad
Third word: bed
Fourth word: bid
Fifth word: cab

Review these five words.

Group 2

First word: bid
Second word: bag
Third word: beg
Fourth word: big
Fifth word: bee

Review these five words.

Group 3

First word: fad
Second word: fed
Third word: fig
Fourth word: fie
Fifth word: fee

Review these five words.

Group 4

First word: gab
Second word: gag
Third word: gee
Fourth word: had
Fifth word: hid

Review these five words.

Group 5

First word: die
Second word: ice
Third word: jab
Fourth word: jag
Fifth word: jig

Review all the words in this group.

* Each word was given three times in succession, with the exception of the fifth word in Group 5, which was given four times.

That concludes Exercise 1 and before proceeding to Exercise 2, once again listen to the tonal alphabet in its entirety. (Alphabet played again.) Exercise No. 2 is made up in same manner as Exercise 1. Emphasis will be on letters from K to T. Ready for Exercise 2.

EXERCISE No. 2.*

Group 1

First word: keg
Second word: kim
Third word: kit
Fourth word: log
Fifth word: lot

Review these five words.

Group 2

First word: men
Second word: map
Third word: mat
Fourth word: mad
Fifth word: moo

Review these five words.

Group 3

First word: nod
Second word: nor
Third word: not
Fourth word: oil
Fifth word: oar

Review these five words.

* Each word was given three times in succession.

Now we shall listen to the alphabet played with neither repetition nor interruption (alphabet in Tonal Braille): and now we turn to a short drill using two pairs of opposites the F, H and J: bad, bed, bid, fad, fig, fee, had, hid, jab, jag, jig.

Now we shall have a few short sentences still concentrating on these two pairs of opposites as well as the other six letters which go to make up the first ten letters of the alphabet.

Dad had a bad headache.
Our chef had a bad cabbage.
He hid a big sheep hide.
Dad jigged a big jig.
Effie aided Jeff hide a bag.

The letter U was played next, followed by these words: quick, q u i c k; quite, q u i t e; us, u s; quilt, q u i l t.

The letters V, W, and X were then played, followed by these words: live, l i v e; move, m o v e; love, l o v e; vail, v a i l; we, w e; was, w a s; saw, s a w; war, w a r; raw, r a w; web, w e b; axle, a x l e.

After playing the letters Y and Z, the lesson concluded with these words: lay, l a y; yes, y e s; my, m y; eye, e y e; only, o n l y; sty, s t y; buzz, b u z z; gave, g a v e; zero, z e r o; zoo, z o o.

# PIECEWISE LINEAR ISOTROPIC PLASTICITY APPLIED TO A CIRCULAR CYLINDRICAL SHELL WITH SYMMETRICAL RADIAL LOADING *

BY

P. G. HODGE, JR.[1]

SUMMARY

A theory of plasticity is developed with specific reference to a circular cylindrical shell subjected to axially symmetric radial loading. The yield condition is expressed in terms of the stress resultants and is represented by a square in stress space. The plastic potential flow law is extended to include the corners of the yield condition, under the assumption that the strain-hardening is isotropic and linear. Next, the flow law at all points is piecewise integrated and expressed as a direct relation between stress and strain but containing certain unknown functions. Under certain restrictions on the loading, the two classical minimum principles of minimum potential and complementary energy are shown to be valid. Finally, these minimum principles are applied to an example and compared with the exact solution.

## 1. INTRODUCTION

The plastic theory of circular cylindrical shells under axially symmetric loading has been the subject of numerous papers in recent years (1–9),[2] so that it would be presumptuous to attempt to make the present paper self contained. Rather, all pertinent definitions and formulas, together with specific references, will be collected in the next section.

Most of the previous work has been concerned with shells made of a perfectly plastic material, the only exception being (9) where a specific problem was solved for a material with linear strain-hardening. In the present paper, shells with linear isotropic strain-hardening will be considered from a somewhat more general viewpoint.

In accordance with the findings of Drucker (10) and Hill (11), the plastic potential flow law (12) will be assumed. This law gives the strain rates as unique functions of the stresses except at the "corners" of the yield curve. The appropriate relations at a corner will be deduced by a limiting process in Section 3, and it will be shown that the same result would be obtained from continuity of the specific power. These results are an extension of those of Koiter (13) and Prager (14). Section 4 shows how the strain-hardening coefficient can be evaluated from a tensile test. In Section 5 it will be shown that the flow law can be "piecewise integrated" to obtain some of the advantages of a deformation theory. Next, the two minimum principles of classical

* The results presented in this paper were obtained in the course of research sponsored by the Office of Naval Research.

[1] Associate Professor of Applied Mechanics, Polytechnic Institute of Brooklyn, Brooklyn, N. Y.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

elasticity will be extended to apply to the present plasticity problem under certain restrictions. The details of the proof will be given in an Appendix. Section 7 will take up a specific application of the minimum principles, and a comparison will be made with the exact solution. Finally, the paper will close with some brief comments on the limitations and possible extensions of the theory.

## 2. BASIC THEORY

Problems in thin shell theory are conveniently described in terms of generalized stresses and strains. Generalized stresses are defined (15) as those resultants which do work. Thus, for a symmetrically loaded circular cylindrical shell without end load, the only resultants to be considered (1, 3) are the circumferential direct stress per unit length $N_\varphi$ and the axial bending moment per unit length $M_x$. It will prove convenient to replace these by dimensionless stresses defined by

$$n = N_\varphi/2Yh, \quad m = M_x/Yh^2, \tag{1a}$$

where $Y$ is the tensile yield stress and the shell dimensions are defined in Fig. 1. In terms of these dimensionless variables the equilibrium equation is (3)

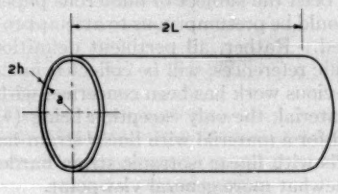



FIG. 1. Dimensions of shell.

$$m''/2c^2 + n + P = 0, \tag{2}$$

where further dimensionless quantities

$$y = x/L, \quad P = pa/2Yh, \quad c = L/\sqrt{ah} \tag{1b}$$

have been defined, $p$ is the exterior pressure, and primes indicate differentiation with respect to $y$.

A necessary ingredient in the description of an isotropic elastic-plastic material is a yield function $f$, with the following properties. If $f$ is less than zero, or if $f$ is less than its previous maximum, or if $f$ is decreasing, then the strain rates are purely elastic; otherwise part of the strain rates may be plastic. In the present paper the function $f$ will be

taken as the simplest approximation to the true yield curve for the shell problem (1, 4),

$$f = \max [|n|, \quad |m|] - 1.$$

The level curves of this function are shown in an $n$, $m$ space in Fig. 2.

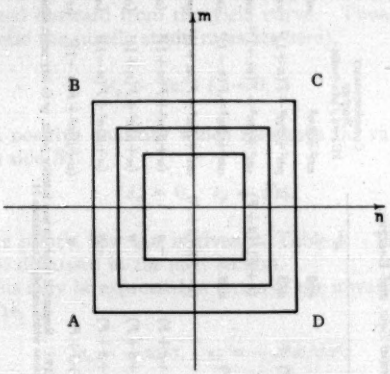



FIG. 2. Level curves of yield function.

In addition to the yield function, it is necessary to give a flow law. This is done in terms of generalized strain rates $e$ and $\kappa$ defined (15) so that the specific power is proportional to

$$D = n\dot{e} + m\dot{\kappa}. \tag{3}$$

Now, if $e_\phi$ and $\kappa_x$ are the unit circumferential extension and axial curvature of the middle section, respectively, the specific power per unit volume is

$$(N_\phi \dot{e}_\phi + M_x \dot{\kappa}_x)/2h.$$

Therefore, in view of Eqs. 1 dimensionless strains may be defined by

$$e = e_x, \quad \kappa = (h/2)\kappa_x. \tag{1c}$$

It is assumed that the total strain rates may be decomposed into an elastic and a plastic part:

$$\dot{e} = \dot{e}_e + \dot{e}_p, \quad \dot{\kappa} = \dot{\kappa}_e + \dot{\kappa}_p. \tag{4}$$

The elastic part will be governed by Hooke's law (16) which may be written

$$\dot{e}_e = \alpha\dot{n}, \quad \dot{\kappa}_e = \beta\dot{m}, \tag{5}$$

TABLE I.—*Flow Law, Stress-Strain Law, and Energies.*

| Regime* | Plastic Flow Law | Stress Strain Law | | | | Complementary Energy $2U_c = 2\int e\,dn + \kappa\,dm$ | Strain Energy $2U = 2\int n\,de + m\,d\kappa$ |
|---|---|---|---|---|---|---|---|
| | | $e$ | $\kappa$ | $n$ | $m$ | | |
| $E$ | $\dot{e}=\alpha\dot{n},\ \dot{\kappa}=\beta\dot{m}$ | $\alpha n$ | $\beta m$ | $e/\alpha$ | $\kappa/\beta$ | $\alpha n^2+\beta m^2$ | $\alpha n^2+\beta m^2$ |
| $AB$ | $\dot{e}_p=F\dot{n},\ \dot{\kappa}_p=0$ | $\alpha n+F(n+1)$ | $\beta m$ | $(e-F)/(\alpha+F)$ | $\kappa/\beta$ | $\alpha n^2+\beta m^2+F(n+1)^2$ | $\alpha n^2+\beta m^2+F(n^2-1)$ |
| $BC$ | $\dot{e}_p=0,\ \dot{\kappa}_p=F\dot{m}$ | $\alpha n$ | $\beta m+F(m-1)$ | $e/\alpha$ | $(\kappa+F)/(\beta+F)$ | $\alpha n^2+\beta m^2+F(m-1)^2$ | $\alpha n^2+\beta m^2+F(m^2-1)$ |
| $CD$ | $\dot{e}_p=F\dot{n},\ \dot{\kappa}_p=0$ | $\alpha n+F(n-1)$ | $\beta m$ | $(e+F)/(\alpha+F)$ | $\kappa/\beta$ | $\alpha n^2+\beta m^2+F(n-1)^2$ | $\alpha n^2+\beta m^2+F(n^2-1)$ |
| $DA$ | $\dot{e}_p=0,\ \dot{\kappa}_p=F\dot{m}$ | $\alpha n$ | $\beta m+F(m+1)$ | $e/\alpha$ | $(\kappa-F)/(\beta+F)$ | $\alpha n^2+\beta m^2+F(m+1)^2$ | $\alpha n^2+\beta m^2+F(m^2-1)$ |
| $A$ | $\dot{e}_p+\dot{\kappa}_p=F\dot{n},\ m=n$ | $e+\kappa=(\alpha+\beta+F)n+F$ | | $(e+\kappa-F)/(\alpha+\beta+F)$ | $-n$ | $(\alpha+\beta)n^2+F(n+1)^2$ | $(\alpha+\beta+F)n^2-F$ |
| $B$ | $\dot{e}_p-\dot{\kappa}_p=F\dot{n},\ m=-n$ | $e-\kappa=(\alpha+\beta+F)n+F$ | | $(e-\kappa-F)/(\alpha+\beta+F)$ | $n$ | $(\alpha+\beta)n^2+F(n+1)^2$ | $(\alpha+\beta+F)n^2-F$ |
| $C$ | $\dot{e}_p+\dot{\kappa}_p=F\dot{n},\ m=n$ | $e+\kappa=(\alpha+\beta+F)n-F$ | | $(e+\kappa+F)/(\alpha+\beta+F)$ | $-n$ | $(\alpha+\beta)n^2+F(n-1)^2$ | $(\alpha+\beta+F)n^2-F$ |
| $D$ | $\dot{e}_p-\dot{\kappa}_p=F\dot{n},\ m=-n$ | $e-\kappa=(\alpha+\beta+F)n-F$ | | $(e-\kappa+F)/(\alpha+\beta+F)$ | $n$ | $(\alpha+\beta)n^2+F(n-1)^2$ | $(\alpha+\beta+F)n^2-F$ |

* The sides and corners of the yield curve have been collectively called plastic regimes (17). Regime $E$ refers to any interior point, or to any point which involves unloading. The flow law is perfectly general but the remainder of the table is valid only for loading conditions which satisfy Conditions 21.

where

$$\alpha = (Y/E), \quad \beta = (3/4)(Y/E)(1 - \nu^2). \tag{1d}$$

The plastic strain rates are given by the plastic potential law (10, 11, 12) which states that the vector with components $e, \kappa$ must be normal to and directed outward from the yield curve. Thus, since $\dot{n} \geq 0$ on side $CD$ (or else the plastic strain rates are zero),

$$\dot{e}_p = F\dot{n}, \quad \dot{\kappa}_p = 0, \tag{6a}$$

where $F$ is a positive quantity which measures the rate of hardening. Similarly, on side $BC$

$$\dot{e}_p = 0, \quad \dot{\kappa}_p = F\dot{m}. \tag{6b}$$

The complete plastic flow law is given in Table I. The flow law at a corner will be discussed in the next section.

The strains may be expressed in terms of the inward radial displacement $w$ by (16, 3)

$$e_x = -w/a, \quad \kappa_x = -d^2w/dx^2.$$

In terms of the dimensionless displacement

$$W = w/a \tag{1e}$$

these may be written

$$e = -W, \quad \kappa = -W''/2c^2. \tag{7}$$

### 3. FLOW LAW AT A CORNER

At the corner $C$ the normal is not defined, hence neither is the flow law. However, a reasonable generalization of the flow law may be obtained by consideration of the limiting case of a curve without corners. A convenient continuous curve is obtained if the corners are replaced by quarter-circles of arbitrarily small radius, Fig. 3. Since the strain-hardening is assumed to be isotropic, the function must be written in such a way that the level curves are similar with respect to the origin. The appropriate equations in the first quadrant are

$$f = \begin{cases} n - 1, & 0 \leq \vartheta \leq \pi/4 - \eta, \\ \{[n - (1 - e)]^2 + [m - (1 - e)]^2 - e^2\} \cdot \{(1 - e)(n + m) \\ \quad - (2 - 4e + e^2) + [(1 - e)^2 (n + m)^2 - (2 - 4e + e^2)(n^2 + m^2)]^{\frac{1}{2}}\}^{-1}, & \pi/4 - \eta \leq \vartheta \leq \pi/4 + \eta, \\ m - 1, & \pi/4 + \eta \leq \vartheta \leq \pi/2, \end{cases} \tag{8}$$

where $\vartheta$, $\epsilon$, and $\eta$ are defined in Fig. 3. It is convenient to rewrite the central portion of this curve in terms of the angle $\varphi$, defined in Fig. 3, so that the central portion of the curve is given parametrically by

$$n = n_0(1 - \epsilon + \epsilon \cos \varphi), \quad m = m_0(1 - \epsilon + \epsilon \sin \varphi). \tag{9}$$

In general, the flow law can be written in the form (10, 12)

$$\dot{e}_p = F(\partial f/\partial n)\dot{f}, \quad \dot{\kappa}_p = F(\partial f/\partial m)\dot{f}. \tag{10}$$

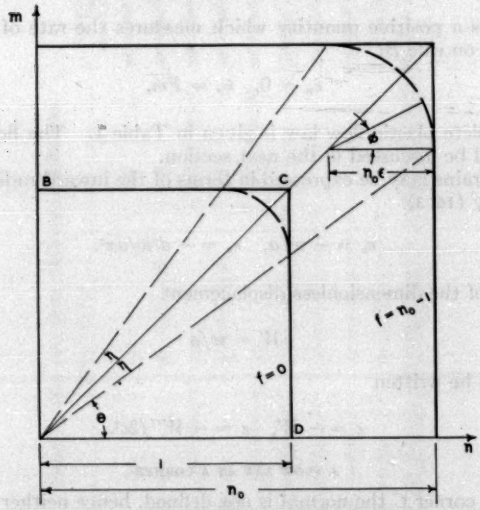



FIG. 3. Level curves for smooth yield function.

——— continuous
– – – – limiting

If the partial derivatives of $f$ are taken from Eqs. 8, $m$ and $n$ are replaced by their values from Eq. 9, and $\epsilon$ is then allowed to tend to zero, the results

$$\partial f/\partial n = \cos \varphi/(\cos \varphi + \sin \varphi),$$
$$\partial f/\partial m = \sin \varphi/(\cos \varphi + \sin \varphi)$$

are obtained. Therefore, the limiting case of the plastic strain rate field may be written

$$\begin{aligned} \dot{e}_p &= F\dot{f}, & \dot{\kappa}_p &= 0, & 0 \le \vartheta < \pi/4, \\ \dot{e}_p &= \frac{F\dot{f}\cos\varphi}{\cos\varphi + \sin\varphi} & \dot{\kappa}_p &= \frac{F\dot{f}\sin\varphi}{\cos\varphi + \sin\varphi} & \vartheta = \pi/4, \\ \dot{e}_p &= 0, & \dot{\kappa}_p &= F\dot{f}, & \pi/4 < \vartheta \le \pi/2. \end{aligned} \tag{11}$$

Finally, $\varphi$ may be eliminated from the second line of Eqs. 11 by adding the two equations there to obtain

$$\dot{e}_p + \dot{\kappa}_p = F\dot{f}, \quad \vartheta = \pi/4. \tag{12}$$

There is only one stress strain relation at the point $C$ since the additional stress equation $m = n$ must also hold there.

A further result of Eqs. 11 is the continuity of the specific power. Indeed, on side $CD$, it follows from Eqs. 2 that

$$D_p = n\dot{F}\dot{f} = Fn\dot{n}, \tag{13a}$$

while on side $BC$,

$$D_p = m\dot{F}\dot{f} = F\,m\,\dot{m}. \tag{13b}$$

Finally, since at the corner $C$, $m = n$ and $\dot{m} = \dot{n}$,

$$D_p = n(\dot{e}_p + \dot{\kappa}_p) = n\dot{F}\dot{f} = F\,n\,\dot{n} = F\,m\,\dot{m} \tag{13c}$$

thus providing continuity with both sides.

Similar computations may be carried out for each side and corner. The resulting plastic flow laws are shown in Table I.

#### 4. DETERMINATION OF $F$

The strain-hardening coefficient $F$ can be determined from a single test, such as a tensile test. To this end, Eq. 13 is rewritten in the form

$$D_p = F(f + 1)\dot{f} \tag{14}$$

which is independent of any coordinates. In simple tension, the significant part of the yield curve is

$$f = \sigma/Y - 1,$$

hence

$$D_p = F\sigma\dot{\sigma}/Y^2. \tag{15}$$

However, $D_p$ is proportional to the specific plastic power, and it is easily shown that

$$D_p = (\sigma/Y)\dot{e}_p. \tag{16}$$

Comparison of Eqs. 15 and 16 and use of the relation $\dot{\epsilon} = \dot{\epsilon}_e + \dot{\epsilon}_p$ then shows that

$$F = Y\, d\epsilon_p/d\sigma = Y(d\epsilon/d\sigma - d\epsilon_e/d\sigma) = Y(1/E - 1/E'), \quad (17)$$

where $E$ is Young's modulus and $E'$ is the tangent modulus (Fig. 4). In the particular case of linear strain-hardening to be considered here, $E'$ is constant, hence $F$ is constant.

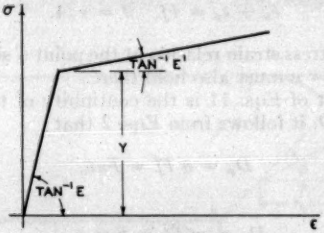



FIG. 4. Tensile stress-strain curve.

### 5. INTEGRATION OF THE FLOW LAW

In view of Eqs. 4 and 5 the complete flow law may be written

$$\dot{e} = \alpha\dot{n} + \dot{e}_p, \quad \dot{\kappa} = \beta\dot{m} + \dot{\kappa}_p \quad (18)$$

where $\dot{e}_p$ and $\dot{\kappa}_p$ are given by the appropriate line of Table I. If a part of the shell remains elastic then Eqs. 18 are trivially integrated:

$$e = \alpha n, \quad \kappa = \beta m. \quad (19a)$$

If an element of the shell goes from the elastic regime into regime $CD$ at a time $t_0$ and thereafter remains there, Eq. 18 must be integrated using the plastic strain rates from line 4 of Table I and Eq. 19$a$ as the initial condition at time $t_0$. Thus

$$e = \alpha n + F(n - 1), \quad \kappa = \beta m. \quad (19b)$$

If an element goes directly from the elastic regime to corner $C$, then

$$\dot{e} + \dot{\kappa} = \alpha\dot{n} + \beta\dot{m} + F\dot{n} = (\alpha + \beta + F)\dot{n}. \quad (20)$$

Since, when $n = 1$, $e = \alpha n = \alpha$ and $\kappa = \beta m = \beta$, integration yields

$$e - \alpha + \kappa - \beta = (\alpha + \beta + F)(n - 1).$$

Finally, then, the stresses are given by

$$n = m = (e + \kappa + F)/(\alpha + \beta + F). \tag{19c}$$

It will be observed that whereas Eqs. 19*b* can be solved for either stress or strain, Eqs. 19*c* can only be solved for stress.

If an element goes first to side $CD$ at $n = 1$ and then to corner $C$ at $n = n_1$, Eqs. 20 must be integrated with initial conditions given by Eqs. 19*b*. Thus

$$e - [\alpha n_1 + F(n_1 - 1)] + \kappa - \beta n_1 = (\alpha + \beta + F)(n - n_1).$$

Evidently $n_1$ drops out of this equation and Eq. 19*c* is valid in this case also. Therefore, if a particle is in a corner, the stress strain law is independent of whether it came there directly or by way of an adjacent side.

Although these are the only types of loadings which will be considered in the present paper, the techniques are easily extended to more complex loading histories and to unloading. However, it must be emphasized that, in general, the final stress strain law is not independent of the loading history. A list of the stress strain laws to be considered is given in Table I.

### 6. STATEMENT OF THE MINIMUM PRINCIPLES

The two minimum principles dealt with in the present section are extensions of the principles of minimum potential energy and minimum complementary energy of classical elasticity. They will be proved under any loadings which satisfy the following conditions.

1. Each point in the shell either remains elastic or becomes and remains plastic.
2. Each point which first becomes plastic at a corner of the yield function remains at that corner.
3. Each point which first becomes plastic at a side of the yield function either remains at that side or moves to and remains at an adjacent corner. (21)

Both the actual state of stress and any artificial states of stress to be considered must satisfy conditions 21 at all stages of loading.

The complementary energy associated with any "statically admissible" state of stress in a cylindrical shell is given by

$$\Pi_c = 2\pi a \int_0^L (e_\varphi \, dN_\varphi + \kappa_x \, dM_x) dx = 4\pi \, a \, h \, L \, Y \, I_c, \tag{22}$$

where

$$I_c = \int_0^1 U_c \, dy, \tag{23}$$

and $U_c$ is the dimensionless unit stress energy,

$$U_c = \int (e\,dn + \kappa\,dm). \tag{24}$$

A statically admissible state is defined as any set of functions $n$, $m$ which are in equilibrium (that is, satisfy Eq. 2) and satisfy any stress boundary conditions, together with functions $e$, $\kappa$ related to $n$, $m$ by the appropriate stress strain law. In general $e$ and $\kappa$ will not be derivable from a displacement field $W$. The principle of minimum complementary energy then states that among all statically admissible states of stress the actual state minimizes the complementary energy $I_c$.

Similarly, the potential energy associated with any "geometrically admissible" state of strain is given by

$$\Pi = 4\pi\, a\, h\, L\, Y\, I, \tag{25}$$

where

$$I = \int_0^1 (U - P\,W)dy, \tag{26}$$

and $U$ is the dimensionless unit strain energy,

$$U = (n\,de + m\,d\,\kappa). \tag{27}$$

A geometrically admissible state is defined as any set of functions $e$, $\kappa$ derived by means of Eqs. 7 from a displacement field $W$ satisfying any displacement boundary conditions, together with functions $n$, $m$ related to $e$, $\kappa$ by the appropriate stress strain law. In general $n$, $m$ will not satisfy equilibrium. The principle of minimum potential energy then states that among all geometrically admissible states of strain the actual state minimizes the potential energy $I$.

The proof of these two principles is given in the Appendix.

The unit stress and strain energies may be integrated for each of the plastic regimes. For example, if a particle goes from the elastic regime to side $CD$ when $n = 1$, and thence to corner $C$ at $n = n_1$,

$$U_c = \int_{n=0}^{n=1} (\alpha\, n dn + \beta m dm) + \int_{n=1}^{n=n_1} [(\alpha\, n + Fn - F)\,dn + \beta\, m\, dm]$$

$$+ \int_{n_1}^{n} (e + \kappa)\,dn = (1/2)[(\alpha + \beta)n^2 + F(n-1)^2]. \tag{28}$$

It is readily verified that Eq. 28 also holds for an element which goes directly to $C$ from the elastic regime.

Similar computations may be carried out for each regime and the results are shown in Table I. The application of the principles will be illustrated by an example in the next section.

### 7. EXAMPLE

As an application of the minimum principles, an unsupported shell which is subjected to a uniform pressure $p$ and to an internal ring load $\lambda Lp$ will be considered. For numerical calculations, the following data will be assumed:

$$c = 2.201, \quad \nu = 0.3, \quad \lambda = 0.1,$$
$$Y/E = 1.75 \times 10^{-3}, \quad E/E' = 400. \tag{29a}$$

It then follows from Eqs. 1*d* and 17 that

$$\alpha = 1.75 \times 10^{-3}, \quad \beta = 1.193 \times 10^{-3}, \quad F = 399\,\alpha. \tag{29b}$$

The coordinate origin will be taken at the center of the shell so that the boundary conditions for a complete solution are

$$W'(0) = 0, \tag{30}$$

$$m'(0) = m(1) = 0, \quad m'(1) = -2c^2\lambda P, \tag{31}$$

where $P$ is the dimensionless load.

Equation 3 for the complementary energy is still valid, but Eq. 6 for the potential energy must be changed to

$$I = \int_0^1 (U - PW)dy + \lambda PW(1) \tag{32}$$

in order to account for the ring load at the ends.

In the application of the theory of minimum potential energy, a displacement function of the form

$$W = A + B\,y^2 \tag{33}$$

which satisfies Boundary Condition 32*a* will be chosen, and $A$ and $B$ are to be determined so that $I$ is a minimum. For $P$ sufficiently small, the solution will be elastic, and it follows from Eqs. 7 and 32 and the elastic entries in Table I that

$$I_e = \int_0^1 (\epsilon^2/2\alpha + \kappa^2/2\beta - P\,W)dy + \lambda P(A + B). \tag{34}$$

If $I$ is minimized with respect to $A$ and $B$ and numerical values substituted from Eqs. 30 the results

$$A = 1.047\,\alpha\,P, \quad B = -0.440\,\alpha\,P \tag{35}$$

are obtained. The substitution of these values into Eqs. 33 and 34 and Table I then gives the assumed solution. The top curve in Fig. 5 shows

the assumed shape for $P = 0.5$, and the dashed curve $P = 0.5$ in Fig. 6 shows "stress profile," that is, the curve in stress space with parametric equations $n = n(y)$, $m = m(y)$.

It is evident from Fig. 6 that the assumed solution is elastic for $P = 0.5$. Further, since $A$ and $B$ are proportional to $P$, the solution

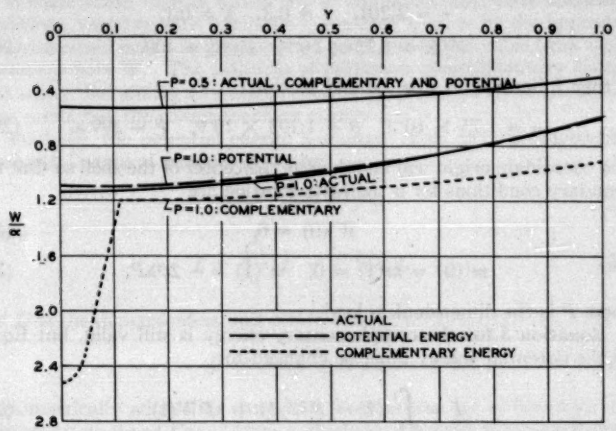



FIG. 5. Deflected shape of shell. $P = 0.5, 1.0$.

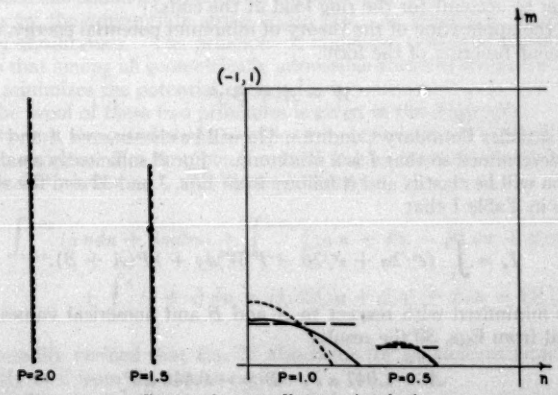



FIG. 6. Stress profiles at various loads.

actual
potential energy
complementary energy

will continue to be elastic with increasing $P$ until the left end of the stress profile meets the side $AB$ of the initial yield curve, that is, until

$$n(0) = -A/\alpha = -1.047\,P = -1, \tag{36}$$

$$P = P^* = 0.955. \tag{37}$$

For $P > P^*$ it will be assumed that the shell is plastic on face $AB$ for $0 < y < \eta$ and elastic for $\eta < y < 1$. The unknown value $\eta$ will be determined so that the stresses are continuous. Thus, from Table I,

$$e(\eta)/\alpha = [e(\eta) - F]/(\alpha + F), \quad e(\eta) + \alpha = 0. \tag{38}$$

After some computation, the potential energy can be written in the form

$$I = I_e - [F/(\alpha + F)] \int_0^\eta (e + \alpha)^2/2\alpha \, dy, \tag{39}$$

where $I_e$ is given by Eq. 34. In taking derivatives of $I$, $\eta$ must be treated as a function of $A$ and $B$, since it is determined by Eq. 38. Thus, for example,

$$\frac{\partial I}{\partial A} = \frac{\partial I_e}{\partial A} - \frac{F}{\alpha(\alpha + F)} \int_0^\eta (e + \alpha) \frac{\partial e}{\partial A} dy - \frac{F}{2\alpha(\alpha + F)} [e(\eta) + \alpha]^2 \frac{\partial \eta}{\partial A}.$$

However, it follows from Eq. 38 that the last term above vanishes, hence the same result is obtained as if $\eta$ were a constant.

The two equations $\partial I/\partial A = 0$, $\partial I/\partial B = 0$, together with Eq. 38 are sufficient to determine $A$, $B$, and $\eta$. However, as is usually the case with such problems, it is more convenient to consider $\eta$ as given and determine the corresponding load $P$. It should be noted that this can only be done after differentiation, since for variational purposes $P$ is constant and $\eta$ is variable. For the numerical example considered,

$$A/\alpha = 1 + B\eta^2/\alpha, \quad B/\alpha = -30/D,$$
$$P = [68.125 + 19.95\eta^3\,(3\eta^2 - 5)]/D, \tag{40a}$$

where

$$D = 71.31 - 30\eta^2 - 1.995\eta^3(35 - 27\eta^2). \tag{40b}$$

The deflected shape is shown in Fig. 7 and the stress profile by the dashed curve in Fig. 6.

As the load continues to increase, the right end of the stress profile in Fig. 6 will move to the left until eventually it reaches the line $n = -1$ at a load which can be determined at

$$P^{**} = 1.116. \tag{41}$$

For $P > P^{**}$ the entire shell is plastic. Minimization of $I$ is easily carried out and leads to

$$A/\alpha = 362.6P - 399, \quad B/\alpha = -1.063P. \tag{42}$$

Deflected shapes and stress profiles are shown by dashed curves in Figs. 6 and 7. Also, the central (maximum) deflection $W(0)$ is shown as a function of $P$ in Fig. 8, while Fig. 9 shows the maximum compressive hoop stress $-n(0)$.

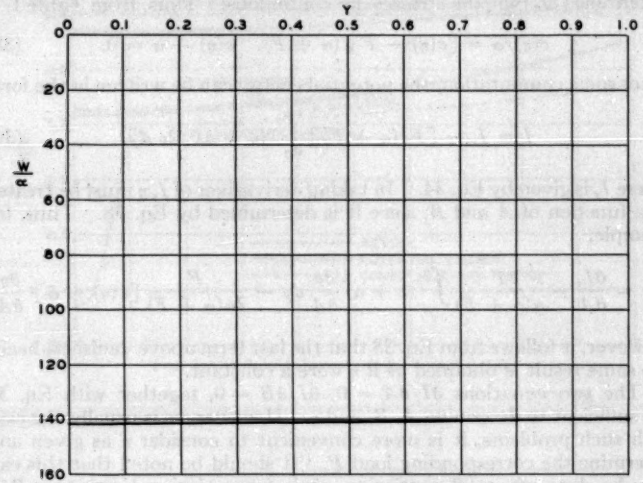



FIG. 7. Deflected shape of shell. $P = 1.5$.
(Same shape predicted by all methods.)

In order to apply the principle of minimum complementary energy, the bending moment is taken to be

$$m = c^2\lambda(1 - y^2)[Py^2 + C(1 - y^2)] \tag{43a}$$

in order to satisfy the boundary conditions 31. It follows from Eq. 2 expressing equilibrium that

$$n = -P(1 + \lambda - 6\lambda y^2) + 2C\lambda(1 - 3y^2). \tag{43b}$$

The complementary energy is then given by Eq. 22 and Table I. When this is minimized with respect to $C$ in the elastic case, the result

$$C = 0.246P \tag{44}$$

is obtained. The top curve in Fig. 5 represents the resulting displacement for $P = 0.5$ in this case also to within the scale used. This is computed from Table I. The stress profile is shown in Fig. 6.

Evidently this solution will remain valid until $n(0) = -1$, and the load at which this occurs turns out to be

$$P_c^* = 0.952. \tag{45}$$

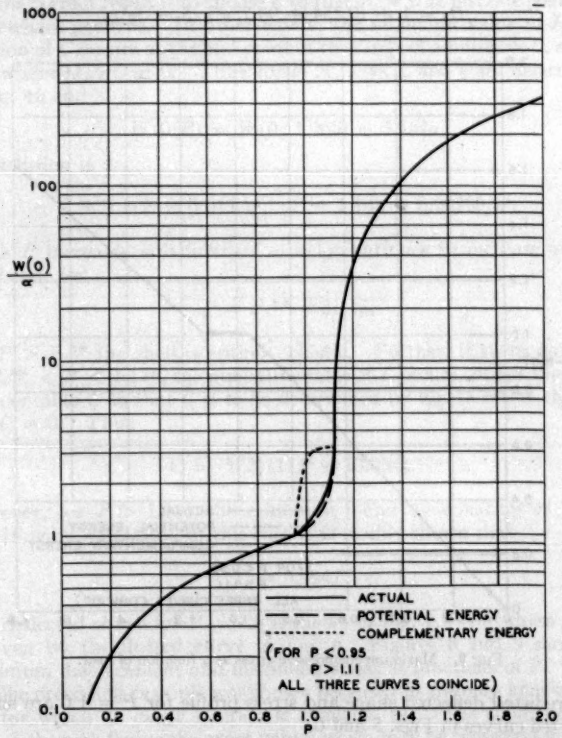



FIG. 8. Maximum displacement as a function of load.

For $P > P_c^*$ the shell is again taken to be plastic on face $AB$ for $0 < y < \eta$ and elastic for $\eta < y < 1$. In this case $\eta$ is determined by

$$n(\eta) = -P(1 + \lambda - 6\lambda\eta^2) + 2C\lambda(1 - 3\eta^2) = -1. \tag{46}$$

Together with the equation $\partial I_c/\partial C = 0$ this provides two relations between $C$, $\eta$, and $P$. For the numerical example these may be written

$$C = (10/9)[1 - 0.254\,(11 - 6\eta^2)/D_c,$$
$$P = (10/9)[1 - 0.254(2 - 6\eta^2)/D_c], \tag{47a}$$

where

$$D_c = 1 + (0.254 + 99.75\eta^3)\,(5 - 3\eta^2). \tag{47b}$$

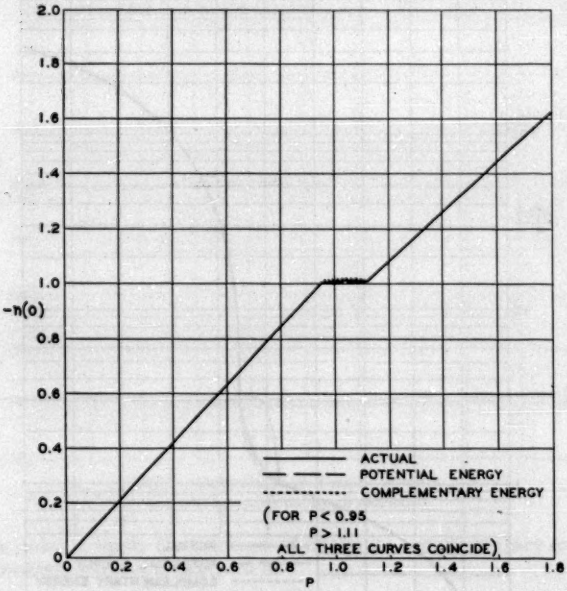



FIG. 9. Maximum compressive stress as a function of load.

The predicted deflected shape and stress profile for $P = 1.0$ are shown by dotted curves in Figs. 5 and 6.

Equations 47 furnish a set of stresses for all values of $\eta$ between zero and one. However, for $P > 1.066$ ($\eta > 0.197$) the resulting displacements, computed from Table I, are found to be decreasing functions of the load. Therefore, the supposed solution is not valid, since Assumptions 21 are not all satified.

A solution for $P > 1.066$ may be obtained in at least three ways:

Eq. 43 may be replaced by a different assumed form for $m$; the assumption that the stress profile is either elastic or on regime $AB$ may be altered; the solution may be made valid by incorporating $\dot{W} \geq 0$ as a hypothesis. This last course will be adopted here, and the analysis proceeds as follows. Considered as a function of $C$, the only extremum and hence the true minimum of $I_e$ occurs when $\dot{W} < 0$. Therefore, if $I_e$ is to be minimized subject to the restriction that $\dot{W} > 0$, it is obvious that the correct result is to choose $C$ so that $\dot{W} = 0$ at just one point and is elsewhere positive. In other words, the minimum value of $I_e$ as a function of $C$ occurs at the end point of the range of admissible $C$, rather than where $\partial I_e/\partial C = 0$. Thus, for $P > 1.066$, $C$ and $\eta$ are determined by Eq. 46 and

$$W(0;P) = W(0; 1.066) = 3.162\alpha. \tag{48}$$

The solution is

$$C = 1.108 - 0.011/\eta^2, \quad P = 1.108 - 0.00200/\eta^2. \tag{49}$$

As $P$ increases still further, $-n(1)$ continues to increase until it equals one for

$$P = P_e^{**} = 1.115. \tag{50}$$

For $P > P_e^{**}$ the shell is entirely plastic. Further, it turns out that for $P_e^{**} < P < 1.116$, the minimum value of $I_e$ occurs outside the range of acceptable $C$ so that $C$ is to be determined by Eq. 48 rather than by $\partial I/\partial C = 0$. Thus

$$C = (1/2)(11P - 10.054). \tag{51}$$

However, for $P > 1.116$ the minimum found by equating $\partial I/\partial C$ to zero is again valid, and in this case it is readily shown that

$$C = 0.994P. \tag{52}$$

The deflected shape for $P = 1.5$ is shown in Fig. 7. The stress profile is given by the dotted curve in Fig. 6. Figures 8 and 9 show the maximum displacement and maximum stress as functions of $P$.

The preceding example was chosen because it is the only known problem for which an exact solution is available (9). The solid curves in Figs. 5 through 9 give the exact values which may be compared with the approximate values given by the two minimum principles.

## 8. CONCLUSIONS

The proceding sections have been concerned with the development of a piecewise linear theory of plasticity as it is applied to circular

cylindrical shells. The development of the theory was carried out in three parts. First, the flow rule at a corner was derived for isotropic strain-hardening. Next, the flow rule was integrated explicitly under certain restrictions contained in Conditions 21, and it was indicated that other cases might be handled similarly. Finally, the minimum principles of classical elasticity were extended to all loading conditions satisfying Conditions 21.

It must be pointed out that the limitations on the minimum principles imposed by Conditions 21 are quite severe and care must be taken that the principles are not misused. Not only must the assumed admissible states satisfy these restrictions, but also the actual state. The former can easily be controlled by choice, but there is no way of knowing whether or not the actual state satisfies the restrictions. The result is that the principles can never be used with the mathematical confidence associated with the theory of elasticity.

Despite this serious limitation, it is believed that the theory proposed here should be of considerable engineering value, since it suggests a method for approximating solutions to exceedingly difficult problems.

For the particular examples considered, it is evident from Figs. 5 through 9 that very close approximations to the true solution can be obtained from simple approximating functions. When the shell is either entirely elastic or entirely plastic, both of the approximate solutions are almost embarrassingly good. If the pressure is such that the shell is partially plastic, then the potential energy still gives a very good approximation to the deflected shape and maximum deflection, while the complementary energy gives reasonable values for the stresses.

In closing, it should be pointed out that the theory is not limited to the particular case of circular cylindrical shells considered here. Investigations are presently under way extending the theoretical results to general three-dimensional elasticity (18) and also applying the minimum principles to determine the true load carrying capacity of a shell subjected to both radial and end load (19).

## REFERENCES

(1) D. C. DRUCKER, "Limit Analysis of Cylindrical Shells under Axially-Symmetric Loading," Proc. 1st Midw. Conf. Solid Mech. (Urbana) pp. 158–163, 1953.

(2) E. T. ONAT, "Plastic Collapse of Cylindrical Shells under Axially Symmetrical Loading," *Q. Appl. Math.*, Vol. 13, pp. 63–72 (1955).

(3) P. G. HODGE, JR., "The Rigid-Plastic Analysis of Symmetrically Loaded Cylindrical Shells," *J. Appl. Mech.*, Vol. 21, pp. 336–342 (1954).

(4) P. G. HODGE, JR., "Impact Pressure Loading of Rigid-Plastic Cylindrical Shells," *J. Mech. Phys. Solids*, Vol. 3, pp. 176–188 (1955).

(5) E. T. ONAT AND W. PRAGER, "Limits of Economy of Material in Shells," *De Ingenieur* ('s Gravenhage), Vol. 67, pp. 0.46–0.49 (1955).

(6) P. G. HODGE, JR., "Displacements in an Elastic-Plastic Cylindrical Shell," *J. Appl. Mech.*, Vol. 23, pp. 73–79 (1956).

(7) P. G. HODGE, JR., "Ultimate Dynamic Load of a Circular Cylindrical Shell," Proc. Sec. Midw. Conf. Solid Mech., pp. 150–177, Purdue Univ., Sept. 1955.
(8) P. G. HODGE, JR., "The Influence of Blast Characteristics on the Final Deformation of Circular Cylindrical Shells," PIBAL Report 266, Polytechnic Inst. of Brooklyn, 1954, to be publ. *J. Appl. Mech.*
(9) P. G. HODGE, JR. AND F. ROMANO, "Deformations of an Elastic-Plastic Cylindrical Shell with Linear Strain-Hardening," *J. Mech. Phys. Solids*, Vol. 4, pp. 145–161 (1956).
(10) D. C. DRUCKER, "A More Fundamental Approach to Plastic Stress-Strain Relations," Proc. 1st U. S. Nat. Congr. Appl. Mech., J. W. Edwards, Ann Arbor, Mich., pp. 487–491, 1952.
(11) R. HILL, "Plastic Distortion of Non-Uniform Sheets," *Phil. Mag.* (7), Vol. 40, pp. 971–983 (1949).
(12) R. v. MISES, "Mechanik der plastischen Formaenderung von Kristallen," *ZAMM*, Vol. 8, pp. 161–185 (1928).
(13) W. T. KOITER, "Stress-Strain Relations, Uniqueness, and Variational Theorems for Elastic-Plastic Materials with a Singular Yield Surface," *Q. Appl. Math.*, Vol. 11, pp. 350–354 (1953).
(14) W. PRAGER, "On the Use of Singular Yield Conditions and Associated Flow Rules," *J. Appl. Mech.*, Vol. 20, pp. 317–320 (1953).
(15) W. PRAGER, "The General Theory of Limit Design," Proc. 8th Inter. Congr. Appl. Mech., Vol. 2, pp. 65–72, 1956, Istanbul, 1952.
(16) S. TIMOSHENKO, "Theory of Plates and Shells," New York, McGraw-Hill Book Co., Inc., 1940.
(17) H. G. HOPKINS AND W. PRAGER, "The Load Carrying Capacities of Circular Plates," *J. Mech. Phys. Solids*, Vol. 2, pp. 1–13 (1953).
(18) P. G. HODGE, JR., "Minimum Principles of Piecewise Linear Isotropic Plasticity," *J. Rat. Mech. Anal.*, to be published.
(19) P. G. HODGE, JR. AND S. V. NARDO, "Carrying Capacity of an Elastic-Plastic Cylindrical Shell with Linear Strain Hardening," PIBAL Rep. 347, Polytechnic Institute of Brooklyn, 1956.

## APPENDIX

### PROOF OF THE MINIMUM PRINCIPLES

In proving the two minimum principles it will be necessary to make use of the principle of virtual work. This may be stated in the form that the external work done by any statically admissible load on any geometrically admissible boundary displacements is equal to the internal work done by the corresponding statically admissible stresses on the geometrically admissible strains. In terms of the present dimensionless quantities this is equivalent to

$$\int_0^1 P^0 W^* dy = \int_0^1 (n^0 \epsilon^* + m^0 \kappa^*) dy. \tag{A1}$$

The two principles are equivalent to stating that the inequalities

$$\Delta I_c = I_c^0 - I_c = \int_0^1 (U_c^0 - U_c) dy \geq 0, \tag{A2}$$

$$\Delta I = I^* - I = \int_0^1 [U^* - U - P(W^* - W)] dy \geq 0, \tag{A3}$$

must be valid. If the principle of virtual work is applied to work done by the difference between the statically admissible and actual states on the actual displacements, the left hand side of Eq. A1 must vanish, since $P$ is everywhere prescribed. Therefore, Inequality A2 may be written

$$\Delta I_c = \int_0^1 [U_c^0 - U_c - (n^0 - n)\epsilon - (m^0 - m)\kappa] dy \geq 0. \tag{A4}$$

The proof of the above inequality is complicated by the fact that the actual and admissible states may be in different regimes. In fact, since a total of nine regimes are considered, there are eighty-one different combinations of actual and admissible regimes. However, due to the symmetry of the yield condition it will not be necessary to consider each of these in detail.

The analysis will be somewhat shortened by separating $\Delta I_c$ into an elastic and plastic part. To this end, plastic quantities are defined by

$$\begin{aligned} U_{cp} &= U_c - (\alpha n^2 + \beta m^2)/2, \\ U_{cp}^0 &= U_c^0 - (\alpha/2)(n^0)^2 - (\beta/2)(m^0)^2, \\ e_p &= e - \alpha n, \qquad \kappa_p = \kappa - \beta m. \end{aligned} \tag{A5}$$

The left hand side of Inequality A4 can then be written

$$\Delta I_c = \int_0^1 Q_c dy + (1/2) \int_0^1 [\alpha(n - n^0)^2 + \beta(m - m^0)^2] dy \tag{A6}$$

where

$$Q_c = U_{cp}^0 - U_{cp} - (n^0 - n)e_p - (m^0 - m)\kappa_p. \tag{A7}$$

Since $\alpha$ and $\beta$ are both positive, the second integrand is non-negative[3] and vanishes if and only if the two states of stress are identically equal. Therefore, a sufficient condition that $\Delta I_c$ be positive is that $Q_c$ be equal to or greater than zero. The proof of this fact will now be divided into various cases.

*1. Actual state elastic.* In this case $e_p$, $\kappa_p$ and $U_{cp}$ all vanish, hence $Q_c = U_{cp}^0$. But from Table I and Eq. A5, $U_{cp}^0$ is equal to one or another of the expressions 0, $F(n \pm 1)^2$, $F(m \pm 1)^2$ and hence is positive.

*2. Actual state on a side.* Without loss of generality the side may be taken as $CD$, hence from Table I and Eq. A5,

$$e_p = F(n - 1),\ \kappa_p = 0, \qquad U_{cp} = (1/2)F(n - 1)^2. \tag{A8}$$

Substitution into Eq. A7 leads to

$$Q_c = U_{cp}^0 + (F/2)n - 1)(n + 1 - 2n^0). \tag{A9}$$

*2a. Admissible state elastic.* Then $U_{cp}^0 = 0$, and $n \geq 1 \geq n^0$ hence $Q_c$ is obviously positive.

*2b. Admissible state on same side* ($CD$). Then $U_{cp}^0 = (1/2)F(n^0 - 1)^2$, hence $Q_c = (F/2)(n^0 - n)^2$ which is positive.

*2c. Admissible state on a neighboring side* ($BC$). Then $U_{cp}^0 = (F/2)(m^0 - 1)^2$, and $Q_c$ can be written $Q_c = (F/2)[(n - m^0)^2 + 2(m^0 - n^0)(n - 1)]$. Since $n \geq 1$ and since on side $BC$ $m^0 \geq n^0$, $Q_c$ is again positive.

*2d. Admissible state on the opposite side* ($AB$). Then $U_{cp}^0 = (F/2)(n^0 + 1)^2$ and $Q_c$ can be put in the form $Q_c = (n + n^0)^2 - 4n^0(n - 1)$. Since $n \geq 1$ and $n^0 \leq -1$, this is certainly positive.

*2e. Admissible state in an adjacent corner* ($C$). Then $U_{cp}^0 = (F/2)(n^0 - 1)^2$, hence $Q_c = (F/2)(n^0 - n)^2$ which is positive.

*2f. Admissible state in an opposite corner* ($B$). Then $U_{cp}^0 = (F/2)(n^0 + 1)^2$. and $Q_c = (n + n^0)^2 - 4n^0(n - 1)$ which is positive.

*3. Actual state in a corner.* Without loss of generality the corner may be taken to be $C$. Further, due to symmetry the admissible state may be taken so that $n^0 \geq m^0$. It then follows from Eq. A5 that

$$e_p + \kappa_p = F(n - 1), \qquad U_c = (F/2)(n - 1)^2, \qquad m = n \tag{A10}$$

[3] For convenience the awkward term "non-negative" will be replaced by the slightly inaccurate word "positive" in the following. It may readily be verified that $\Delta I_c$ can vanish only if $n^0 = n$, $m^0 = m$ throughout the shell so that the minimum is unique.

so that Eq. A7 may be written

$$Q_c = U_{cp}^0 - (F/2)(n-1)^2 + (n-n^0)(e_p+\kappa_p) + (n^0-m^0)\kappa_p$$
$$= U_{cp}^0 + (F/2)(n-1)(n+1-2n^0) + (n^0-m^0)\kappa_p. \tag{A11}$$

Since a particle at $C$ either came there directly or by way of sides $BC$ or $CD$, $\kappa_p$ was equal or greater than zero during the entire loading history, hence $\kappa_p \geq 0$. The last term in Eq. A11 is thus positive, since $n^0 \geq m^0$ by hypothesis. Further, the remaining terms are identical with Eq. A9 for $Q_c$ under Case 2. Therefore, since as far as any inequalities involving stresses are concerned, Case 3 is merely a special case of Case 2, the proof for Case 2 is applicable, and $Q_c \geq 0$.

To prove the principle of minimum potential energy the principle of virtual work is applied to the work done by the actual stresses on the difference between the geometrically admissible and the actual displacements. Inequality A3 may then be written

$$I = \int_0^1 [U^* - U - n(e^* - e) - m(\kappa^* - \kappa)]dy \geq 0. \tag{A12}$$

Here again it is convenient to separate the elastic and plastic contributions by defining

$$U_p = U - (\alpha n^2 + \beta m^2)/2, \qquad U_p^* = U^* - (\alpha n^{*2} + \beta m^{*2})/2$$
$$e_p = e - \alpha n, \qquad e_p^* = e^* - \alpha n^*$$
$$\kappa_p = \kappa - \beta m, \qquad \kappa_p^* = \kappa^* - \beta m^*. \tag{A13}$$

Then,

$$\Delta I = \int_0^1 Q dy + \int_0^1 [(\alpha/2)(n-n^*)^2 + (\beta/2)(m-m^*)^2]dy \tag{A14}$$

where

$$Q = (U_p^* - ne_p^* - m\kappa_p^*) - (U_p - ne_p - m\kappa_p). \tag{A15}$$

Again, since the last term in Eq. A14 is positive it is sufficient to prove $Q$ is positive for any combination of actual and admissible regimes. The situation is slightly different in that the first parenthesis in Eq. A15 must be evaluated according to the admissible stress strain law, the second according to the actual. The resulting expressions for $Q$ in each case are shown in Table II and it is evident that $Q$ is positive so that the principle has been proved.

TABLE II.—*Potential Energy Kernel.*

| Actual Regime | Admissible Regime | $2Q/F$ | Remarks |
|---|---|---|---|
| $E$ | $E$ | $0$ | |
| $E$ | $CD$ | $(n^*-1)(n^*+1-2n)$ | $n^* \geq 1 \geq n$ |
| $E$ | $C$ | $(n^*-1)(n^*+1-2n)+2(n-m)\kappa_p^*$ | $\kappa_p^* \geq 0$. Use if $n^* \geq 1 \geq n \geq m$ |
| | | $=(n^*-1)(n^*+1-2m)+2(m-n)e_p^*$ | $e_p^* \geq 0$. Use if $n^* \geq 1 \geq m \geq n$ |
| $CD$ or $C$ | $E$ | $(n-1)^2$ | |
| $CD$ or $C$ | $CD$ | $(n-n^*)^2$ | |
| $CD$ or $C$ | $BC$ | $(m^*-n)^2+2(n-m)(m^*-1)$ | $n \geq m$, $m^* \geq 1$ |
| $CD$ or $C$ | $AB$ | $(n^*+n)^2-4n(n^*+1)$ | $n \geq 0$, $n^* \leq -1$ |
| $CD$ or $C$ | $C$ | $(n-n^*)^2+2(n-m)\kappa_p^*$ | $n \geq m$, $\kappa_p^* \geq 0$ |
| $CD$ or $C$ | $B$ | $(n^*+n)^2-2(n-m)(n^*+1)-2(n+m)e_p^*$ | $n \geq m \geq 0$, $n^* \leq -1$, $e_p^* \leq 0$ |
| $CD$ or $C$ | $A$ | $(n^*+n)^2-2(n+m)(n^*+1)-2(n-m)e_p^*$ | $n \geq m \geq 0$, $n^* \leq -1$, $e_p^* \leq 0$ |

**120-Hr. Mission.**—Volunteers of the Air Research and Development Command (ARDC) will spend five days and nights in a confined, grounded aircraft cabin as a test of their efficiency and endurance under long-range flight conditions.

Planned for early February, the test will be conducted by ARDC's Wright Air Development Center (WADC) to determine how a five-man flight crew would react, both mentally and physically, to prolonged flight. An enclosed airplane cabin, in which the men will eat, sleep and "fly" together for 120 consecutive hours, is currently being constructed in the Experimental Fabrication Division shops at WADC.

The "flight" will be made under actual mission conditions. Each of the five men, all pilots with long-range mission experience, will sit in individual, emergency escape capsules when working. The cabin will also contain a recreation and rest area equipped with food bar and sanitary facilities, as well as a working area.

The aircrew will be divided into two groups. The pilot, co-pilot and engineer will be interchangeable so that one can rest while the other two work, thus a pilot and engineer will always be on duty. The other two men, the navigator and defense director, also will be interchangeable, with one serving as navigator, on duty while the other rests.

In the case of an "emergency" (simulated by a smoke bomb in the air conditioning system, a change in the instrument readings or merely by the announcement of an emergency) all men will take their places in their escape capsules, and the panel instruments in all five stations will operate.

While one of the primary objectives of the test will be to determine the ability of five men to work and live together continually, studies also will be made of their efficiency during those five days and nights. This will be accomplished by the numerous tasks they will be asked to perform while working and during "emergencies."

For instance, the men on duty will watch for lights to appear on their instrument panels. To keep them extinguished they will flip switches or adjust knobs. Timing units will tick continually. Whenever a dial moves a unit, the men must press a button. Their reaction times will be measured, both as to how quickly they determine that some action must be taken and how quickly they take action.

The volunteers will be "wired" for brain, heart, and skin response readings. They will be photographed by eight motion picture cameras and watched by two closed-circuit television cameras. All conversations between subjects and test operators will be tape recorded.

The cabin has been designed for maximum comfort. Padded bulkheads and capsules will make it unnecessary to wear helmets or parachutes. Each man will have a choice of three programs of recorded music piped into the communication system.

To provide a feeling of spaciousness, all cabin furnishings will be kept at shoulder height. This spaciousness is further enhanced by the dark floors and lighter walls and ceilings. Indirect lights will be used with switches to change the colors from cool to warm and an illusion of night flight can be created by dimming the lights.

The 120-hr. test, a continuation of previous human endurance tests of 24 hr. and 56 hr., is a project of the Stress and Fatigue Section of the WADC Aero Medical Laboratory. Project director is Mr. Charles A. Dempsey, who has been a volunteer subject during the previous tests.

# MECHANICAL PROPERTIES OF MATERIALS FOR COMBINED STRESSES BASED UPON TRUE STRESS AND STRAIN

BY

JOSEPH MARIN [1]

SUMMARY

In this paper mechanical properties are first defined for simple tension based upon true stress and true strain. An expression for the true toughness for simple tension is obtained in terms of the strain hardening exponent and the strength coefficient. Based upon an assumed true stress-strain relation in simple tension, expressions for yield strength, ultimate strength, ductility and toughness for combined states of stress are found. In formulating the "true" mechanical properties for combined stresses, the simple deformation theory of plasticity is used for expressing the plastic combined stress-strain relations. The use of true stress-strain properties is of importance in selecting materials since the conventional nominal properties may not indicate the correct selection.

## 1. SIMPLE STATIC STRESSES—MECHANICAL PROPERTIES

For simple tension the conventional procedure is to define the mechanical properties based on a nominal stress-strain diagram as indicated in Fig. 1*a*. A more accurate measure of the properties can be obtained based on the true stress-strain relation shown in Fig. 1*b*. In Fig. 1*b* the true stress $\sigma$ and true strain $\delta$ are based on the actual values of the area $A$ of the specimen and the actual gage length $L$ rather than on the original values $A_0$ and $L_0$. It can then be shown that (1)[2]

$$\sigma = \frac{P}{A} \qquad \delta = \int_{L_0}^{L} \frac{dL}{L} = \log_e \frac{L}{L_0}. \tag{1}$$

Based on the "true" properties rather than the nominal values it is recommended that:

1. *The true ultimate stress* $\sigma_u$ be used as a measure of ultimate strength in place of the nominal ultimate stress $\sigma_u'$ or nominal fracture stress $\sigma_f'$.
2. *The true strain* $\delta_u$ (true strain corresponding to $\sigma_u'$) be used as a measure of ductility in place of the usual nominal fracture strain $e_f$.
3. *The true toughness* $T$ (as measured by the area under the true stress-strain diagram to $\sigma_u$) be used in place of the conventional modulus of toughness $T'$ (as measured by the area under the nominal stress-strain diagram).

[1] Professor and Head, Department of Engineering Mechanics, Pennsylvania State University, University Park, Pa.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

The use of the true values $\sigma_u$, $\delta_u$ and $T$ (Fig. 1*b*) in place of $\sigma_u'$, $\epsilon_f$ and $T'$ (Fig. 1*a*) is recommended not only because the true values are an accurate measure of the stress and strain, but also because for design purposes the part of the stress-strain curve between $B_1$ and $C_1$ or $B$ and $C$ in Fig. 1*b* represents a failure condition for the material.

Many investigations have been conducted in which true stress-strain relations have been obtained, as for example, those reported by Low

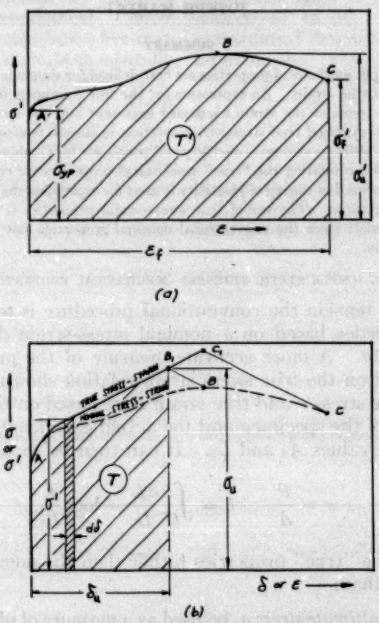



FIG. 1. Nominal and true stress-strain curves.

and Garafalo (2). These results and others show that when the true stress and true strain are plotted on a log-log basis the results may be approximately represented by a straight line (Fig. 2). That is,

$$\log \sigma = n \log \delta + \log k \tag{2}$$

$$\sigma = k\, \delta^n \tag{3}$$

where $k$ is referred to as the *strength coefficient* and $n$ as the *strain hardening exponent.*

The strain $\delta$ in Eq. 3 is usually taken as the plastic strain only. It has been found (3) that the complete true stress-strain diagram may be represented by considering the elastic strain component in writing Eq. 3. Then the total strain is

$$\delta_T = \delta + \delta_E \tag{4}$$

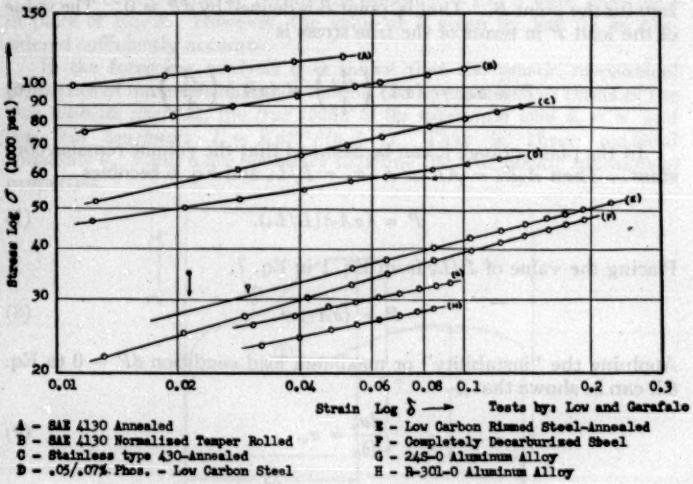



FIG. 2. Logarithmic stress-strain curves for a number of different sheet materials.

where $\delta_E = \dfrac{\sigma}{E}$ = the elastic strain. Then by Eq. 3,

$$\sigma = k\left(\delta_T - \frac{\sigma}{E}\right)^n$$

or

$$\delta_T = \frac{\sigma}{E} + q\,\sigma^m \tag{5}$$

where $q = 1/k$ and $m = 1/n$.

Many attempts (4) have been made during the past hundred years to express the stress-strain relation for simple tension. Most of these, however, have been based on the nominal stress and strain. For many materials it appears that Eq. 5 is a good approximation for the true stress-strain relation and may be used as a basis for stress analysis and design in the plastic range.

For many materials the determination of the ultimate stress point $B$ in Fig. 1*a* is difficult because the stress-strain curve is flat in the region of $B$. For this reason it is difficult to obtain an accurate value of $\delta_u$. A method of more accurately determining $\delta_u$ consists in defining the maximum load point $B$ theoretically, by noting that the load $P$ is represented by the condition that the slope of the $P - \epsilon$ or $\sigma' - \epsilon$ curve (Fig. 1*b*) is zero for the point $B$. That is, point $B$ is defined by $dP = 0$. The value of the load $P$ in terms of the true stress is

$$P = \sigma A = (\sigma A)\left(\frac{A_0}{A_0}\right) = (\sigma A_0)\left(\frac{A}{A_0}\right). \tag{6}$$

In the plastic range it can be assumed that the volume remains constant. Then $A_0L_0 = AL$ or $A/A_0 = L/L_0$ and Eq. 6 becomes

$$P = (\sigma A_0)(L/L_0). \tag{7}$$

Placing the value of $L/L_0$ from Eq. 1 in Eq. 7,

$$P = (\sigma A_0)\, e^{\delta}. \tag{8}$$

Applying the "instability" or maximum load condition $dP = 0$ to Eq. 8 it can be shown that

$$\frac{d\sigma_u}{d\delta_u} = \sigma_u. \tag{9}$$

If the stress as given by Eq. 3 is placed in Eq. 9, then

$$\delta_u = n, \tag{10}$$

that is, the true strain $\delta_u$ at the maximum load is equal to the strain hardening exponent. In other words, the strain hardening exponent can be used as a measure of *true ductility*. A survey (5) of five investigations of 31 materials for which 180 true stress-strain relations were obtained shows that Eq. 10 is a good approximation. The average deviation between actual values $\delta_u$ and theoretical values $n$ was found to be only 2 per cent.

An expression for the *true toughness* $T$, as measured by the shaded area in Fig. 1*b*, can be obtained if the true stress-strain relation of Eq. 3 is used. That is, the area under the true stress-strain diagram to point $B_1$ in Fig. 1*b* is

$$T = \int_0^{\delta_u} \sigma d\delta. \tag{11}$$

Placing $\sigma = k\,\delta^n$ in Eq. 11, integrating, and noting that $\delta_u = n$, Eq. 11 becomes

$$T = \frac{(\sigma_u)^{1+1/n}}{k^{1/n}(n+1)} = \frac{k n^{n+1}}{n+1}. \tag{12}$$

A more accurate value of $T$ could be obtained if Eq. 5 were used for $\delta$ in place of Eq. 3. However, the value of $T$ given by Eq. 12 is considered sufficiently accurate.

In the foregoing analysis it is shown that the plastic mechanical properties of materials in simple tension can be expressed in terms of the *true ultimate stress* $\sigma_u$, the *true strain at the maximum load* $\delta_u = n$, and the *true toughness* $T = k\,n^{n+1}/(n+1)$. That is, three material constants $\sigma_u$, $k$ and $n$ are necessary to define the true plastic mechanical properties.

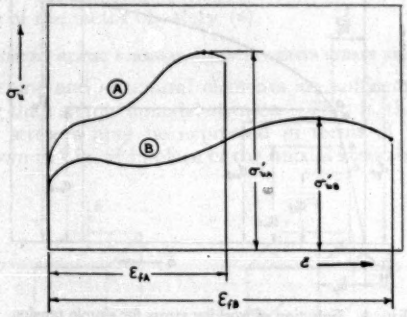



FIG. 3. Comparison of stress-strain curves.

True stress-strain plastic properties can also be obtained for simple compression. However, few relations have been reported so that the validity of Eq. 3 or 5 for simple compression has not been established.

In accepting true stress-strain measures of the plastic properties it may be argued that those properties are only to be used in a qualitative way for selecting materials and that for this purpose the usual nominal properties are adequate. This argument is of course incorrect since it is quite possible that a material selected based on nominal properties will have a different rating than when based on true properties.

Equation 3 or 5 can be used to obtain stress and deformation relations for members subjected to simple stresses such as bending and torsion. These relations can then be used as a basis for the design of such members.

### 2. SIMPLE STATIC STRESSES—DETERMINATION OF WORKING STRESS

The selection of a design or allowable stress is usually determined by dividing the yield stress $\sigma_{yP}$ or the nominal ultimate stress $\sigma_u'$ by an appropriate factor of safety. That is,

$$\sigma_W = \frac{\sigma_y'^P}{N_y} \tag{13a}$$

or

$$\sigma_W = \frac{\sigma_u'}{N_u}. \tag{13b}$$

It is realized that Eqs. 13 do not take into account the relative values of ductility. For example, consider two materials as shown in Fig. 3—one

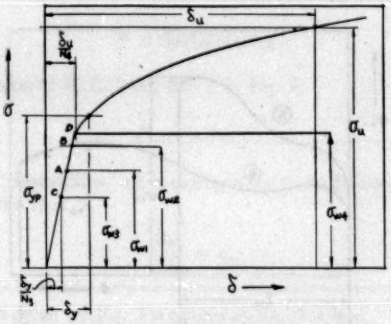



FIG. 4. Selection of working stress for simple tension.

a high strength steel alloy (A) and the other a low strength structural steel (B). Then, according to Eq. 13*b*, if the same value of $N_u$ is used the working stresses for materials (A) and (B) are, respectively,

$$\sigma_{WA} = \frac{\sigma_{uA}'}{N_u} \quad \text{and} \quad \sigma_{WB} = \frac{\sigma_{uB}'}{N_u}. \tag{14}$$

In evaluating the working stresses, no provision is made for the fact that material B has a larger strain at fracture or at the ultimate load than material A. This larger plastic deformation gives material B an advantage over material A which is not reflected in Eqs. 14. To provide for this oversight and to give a more accurate quantitative measure of the working stress, a new method of selecting this stress is suggested.

Referring to the true stress-strain diagram of a material, as shown in Fig. 4, four working stresses will be defined—two based on the yield

stress and true ultimate stress and two based on the strain at the yield stress and the strain at the true ultimate stress. That is, referring to Fig. 4 and assuming the stress-strain relation defined by Eq. 3,

$$\left.\begin{aligned} \sigma_{W_1} &= \frac{\sigma_{yp}}{N_1} \qquad \sigma_{W_2} = \frac{\sigma_u}{N_2} \\ \sigma_{W_3} &= \frac{E\delta_y}{N_3} \qquad \sigma_{W_4} = \left(\frac{k}{N_4}\right)(\delta_u)^n \end{aligned}\right\} \tag{15}$$

Equations 15 define the four working stress values indicated by points $A$, $B$, $C$ and $D$ in Fig. 4. The governing working stress would then be the smallest of these stresses. In the case of Fig. 4 the stress selected would be $\sigma_{W_3}$ as defined by point $C$.

The selection of the factors of safety $N$ in Eqs. 15 is based on many considerations. Various attempts have been made to provide a rational determination of the factor of safety (6).

### 3. COMBINED STATIC STRESSES—PLASTIC STRESS-STRAIN RELATIONS

Many machine and structural elements are subjected to combined stresses rather than simple uniaxial stresses. That is, the state of stress for combined stresses may be expressed in terms of three principal stresses as shown in Fig. 5*b* in place of the unixial stress of Fig. 5*a*. The

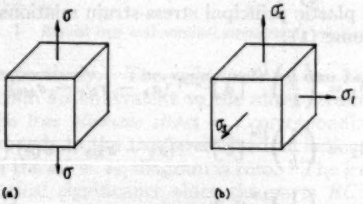



FIG. 5. Triaxial stresses.

principal stresses act on three mutually perpendicular planes on an element. One of these stresses $\sigma_1$ is the maximum possible value of the normal stress, the other $\sigma_3$ is the minimum possible normal stress value for the point considered, and the third $\sigma_2$ is intermediate in value between $\sigma_1$ and $\sigma_3$.

Referring to Fig. 5*b*, if $\delta_1$, $\delta_2$ and $\delta_3$ are the principal strains in the direction of the principal stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$, then the mechanical properties for combined stresses can be obtained if the stress-strain relations to failure are known for each principal stress. It is apparent that there are many of these principal stress-strain relations since the relations depend upon the values of principal stress ratios $\sigma_2/\sigma_1$ and $\sigma_3/\sigma_1$.

Some of these possible relations are indicated in Fig. 6. Because of the numerous principal stress-strain relations possible, and owing to the difficulty of obtaining these relations experimentally, many attempts have been made to predict theoretically the principal stress-strain relations from the stress-strain relation in simple tension (7, 8, 9, 10). For conditions in which the principal stress ratios remain constant during loading, the simple deformation theory (7) has been found to provide a good approximation for defining the plastic stress-strain relations under combined stresses. If the plastic stress-strain relation is defined by

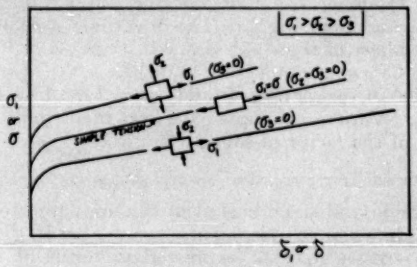



FIG. 6. Biaxial true stress-strain relations.

Eq. 3, then the plastic principal stress-strain relations by the deformation theory become (1):

$$\delta_1 = \left(\frac{1}{k}\right)^{1/n} [\bar{\sigma}]^{1/n-1} (\sigma_1 - \sigma_{2/2} - \sigma_{3/2})$$

$$\delta_2 = \left(\frac{1}{k}\right)^{1/n} [\bar{\sigma}]^{1/n-1} (\sigma_2 - \sigma_{3/2} - \sigma_{1/2}) \qquad (16)$$

$$\delta_3 = \left(\frac{1}{k}\right)^{1/n} [\bar{\sigma}]^{1/n-1} (\sigma_3 - \sigma_{1/2} - \sigma_{2/2})$$

where

$$\bar{\sigma} = \sqrt{\tfrac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}. \qquad (17)$$

By the first of Eqs. 16 the biaxial $\sigma_1 - \delta_1$ relations shown in Fig. 6 can be determined from the simple tension relation. It should be noted that the experimental verification of the theoretical stress-strain relations given in Eq. 16 has been obtained for only a few materials and primarily for the biaxial states of stress shown in Fig. 6. This lack of experimental verification is understandable when it is realized that combined stress experiments are difficult to perform. In fact, for some stress combinations, suitable experiments have not been devised.

#### 4. COMBINED STATIC STRESSES—MECHANICAL PROPERTIES

Figure 7 shows a principal true stress-strain diagram for any stress combination, as obtained by experiment. In plotting Fig. 7 the elastic strains $\delta_{E_1}$ have been added to the plastic strains $\delta_1$ to give the total strains $\delta_{T_1}$. That is,

$$\delta_{T_1} = \delta_{E_1} + \delta_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2 + \sigma_3) + \delta_1. \tag{18}$$

Points $A$, $B$ and $C$ correspond respectively to the yield stress, true ultimate stress (or stress at the maximum load or instability), and true fracture stress. These stresses will be represented by the symbols

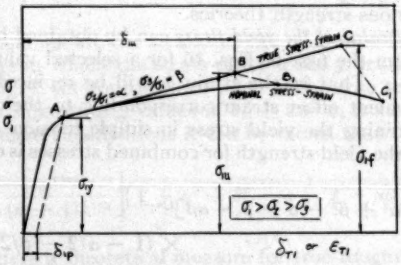



FIG. 7. Biaxial true and nominal stress-strain relation.

$\sigma_{1y}$, $\sigma_{1u}$ and $\sigma_{1f}$, respectively. The *yield stress* $\sigma_{1y}$ can be determined by using an offset strain $\delta_{1P}$ equivalent to the offset strain used in simple tension (11). The *true ultimate stress* $\sigma_{1u}$, corresponding to the maximum load, corresponds to the maximum load as indicated by point $B_1$ where the slope of the $\sigma_1' - \epsilon_{T_1}$ diagram is zero. The *true fracture stress* $\sigma_{1f}$ is not of practical significance since the parts $BC$ or $B_1C_1$ of the stress-strain diagrams represent a region of changing state of stress produced by significant changes in dimensions.

*A theoretical value of the true ultimate stress* (at maximum load or instability) can be found for specific members and types of loading based on the deformation theory as defined by Eqs. 16. That is, the value of ultimate strength as measured by $\sigma_{1u}$ can be obtained from the first of Eqs. 16 provided the value of $\sigma_{1u}$ corresponding to $\delta_{1u}$ is known. That is, from the first of Eqs. 16 the ultimate strength is defined as

$$(\sigma_{1u})^{1/n}\left[1 + \alpha^2 + \beta^2 - \alpha - \beta - \alpha\beta\right]^{(1-n)/2n}(1 - \alpha/2 - \beta/2) = \left(\frac{\delta_{1u}}{\delta_u}\right)(\sigma_u)^{1/n}. \tag{19}$$

The value of $\delta_{1u}$ depends upon the type of loading and member. For example, in a thin-walled spherical shell subjected to internal pressure, using Eqs. 16, the theoretical value of $\delta_{1u} = \delta_{2u}$ can be shown to be $\delta_{1u} = n/3$ (12, 13). Placing values of $\delta_1 = \delta_{1u} = n/3$, $\sigma_2 = \sigma_{1u}$, $\sigma_1 = \sigma_{1u}$, $\sigma_3 = 0$ in the first of Eqs. 16, it can be shown that

$$\sigma_{1u} = (\tfrac{2}{3})^n \sigma_u \tag{19a}$$

where $\sigma_u$ = the true ultimate stress (at maximum load) in simple tension.

Most values of $n$ lie approximately between 1/8 and 1/4. For these values the strength $\sigma_{1u}$ is 0.95 $\sigma_u$ and 0.91 $\sigma_u$, respectively. This corresponds approximately to the value of $\sigma_{1u} = \sigma_u$ which is obtained using most of the various strength theories.

*A theoretical value of the yield stress* can be obtained by determining the stress $\sigma_1$ from the first of Eqs. 16 for a selected value of the offset plastic strain $\delta_1$. That is, the strain $\delta_1$ will be replaced by $\delta_{1P}$ where $\delta_{1P}$ is the equivalent offset strain corresponding to the offset strain $\delta_P$ used for determining the yield stress in simple tension. Then by the first of Eqs. 16 the yield strength for combined stresses is defined by

$$\left(\frac{\sigma_{1y}}{k}\right)^{1/n} [1 + \alpha^2 + \beta^2 - \alpha - \beta - \alpha\beta]^{1/n-1} \times (1 - \alpha/2 - \beta/2) = \delta_{1P}. \tag{20}$$

The value of $\delta_{1P}$ selected will be based on the ratio of the strains $\delta_{1u}$ to $\delta_u$ for the particular principal stress ratio. That is,

$$\delta_{1P} = Q\delta_P \tag{21}$$

where $\delta_P$ = the offset strain for simple tension, and $Q = \delta_{1u}/\delta_u$.

For the case of the thin-walled spherical shell subjected to internal pressure, $Q = \delta_{1u}/\delta_u = 1/3$, $\alpha = 1$, $\beta = 1$ and, by Eqs. 20 and 21,

$$\sigma_{1y} = k(2Q\delta_P)^n = [2\left(\frac{\delta_{1u}}{\delta_u}\right)(\delta_P)]^n(k) = (\tfrac{2}{3}\,\delta_P)^n k. \tag{22}$$

For a particular material, $\sigma_{1y}$ can be determined from Eq. 22 since $k$, $n$ and $\delta_P$ are known.

The *true ductility* can be measured by the maximum normal strain corresponding to the maximum load or point of instability. Since $\sigma_{1u}$ is the maximum stress, the maximum normal strain at instability or true ductility will be designated by $\delta_{1u}$ (Fig. 7). It is apparent that the value of $\delta_{1u}$ will depend upon the principal stress ratios $\alpha = \sigma_2/\sigma_1$ and $\beta = \sigma_3/\sigma_1$ and that these stress ratios should be specified when values of $\delta_{1u}$ are given. Theoretical values of $\delta_{1u}$ can be found, based on the first

of Eqs. 16, for particular states of stress and members (12, 13). For example, for a thin-walled spherical shell under internal pressure the strains at the maximum pressure are $\delta_{1u} = \delta_{2u} = n/3$.

An experimental value of *true toughness* can be obtained by evaluating the work done in a unit volume in stressing to the maximum load or point of instability as represented by the areas under the $\sigma_1$, $\sigma_2$ and $\sigma_3$ true stress-strain diagrams to the stress corresponding to the maximum load. A theoretical value of the true toughness can be obtained based on Eqs. 16. Since the strain energy for the elastic range is small it can be neglected and the true toughness then becomes

$$T = \int_0^{\delta_{1m}} \sigma_1 d\delta_1 + \int_0^{\delta_{2m}} \sigma_2 d\delta_2 + \int_0^{\delta_{3m}} \sigma_3 d\delta_3 \qquad (23)$$

where $\delta_{1m}$, $\delta_{2m}$ and $\delta_{3m}$ are the strains at the maximum load.

For loadings in which the stress ratios $\alpha$ and $\beta$ remain constant, as indicated in Fig. 7, an approximate value of the true toughness can be determined by substituting values of $\delta_1$, $\delta_2$ and $\delta_3$ from Eqs. 16 in Eq. 23. Making these substitutions it can be shown that

$$T = \left[\frac{\sigma_{1u}^{1/n+1}}{(n+1)k^{1/n}}\right][1 + \alpha^2 + \beta^2 - \alpha - \beta - \alpha\beta]^{(1+n)/2n}. \qquad (24)$$

Equation 24 gives a theoretical measure for true toughness for the case of triaxial stresses in which the principal stress ratios $\alpha$ and $\beta$ remain constant.

## CONCLUSION

Mechanical properties of materials based upon true stress and strain are defined in this paper for both simple and combined stresses. Mechanical properties based upon the usual nominal properties may not be adequate for comparing properties of materials. That is, the rating of a material based upon true ductility may be quite different from its rating based upon the usual nominal ductility as measured by the percentage elongation in a tension test.

## REFERENCES

(1) J. MARIN, "Engineering Materials—Their Mechanical Properties and Applications," New York, Prentice-Hall Book Co., 1952.

(2) J. LOW AND F. GARAFALO, "Precision Determination of Stress-Strain Curves in the Plastic Range," *Proc. SESA*, Vol. 4, pp. 16–25 (1947).

(3) F. B. STULEN, Private Communication. Report of the Propeller Division, Curtiss-Wright Corp., Caldwell, N. J.

(4) W. R. OSGOOD, "Stress-Strain Formulas," *J. Aero Sciences*, Vol. 13, p. 45 (1946).

(5) J. MARIN, "A New Criterion for Ductility," report prepared for Curtiss-Wright Corp.

(6) J. MARIN, "Working Stresses," *Machine Design*, 1942.

(7) A. NADAI, "Theory of Flow and Fracture of Materials," New York, McGraw-Hill Book Co., Inc., Vol. 1, 1950.
(8) R. HILL, "The Mathematical Theory of Plasticity," Oxford Univ. Press, 1950.
(9) W. PRAGER AND P. G. HODGE, "Theory of Perfectly Plastic Solids," New York, John Wiley and Sons, Inc., 1951.
(10) O. HOFFMAN AND G. SACHS, "Introduction to the Theory of Plasticity for Engineers," New York, McGraw-Hill Book Co., Inc., 1953.
(11) J. MARIN, B. H. ULRICH AND W. P. HUGHES, "Plastic Stress-Strain Relations for 75S-T6 Aluminum Alloy Subjected to Biaxial Tensile Stresses," NACA Tech. Note 2425, 1951.
(12) G. SACHS AND J. D. LUBAHN, "Failure of Ductile Metals in Tension," *Trans. A.S.M.E.*, Vol. 68, pp. 271–276 (1946).
(13) W. T. LANKFORD AND E. SAIBEL, "Some Problems in Unstable Plastic Flow under Biaxial Tension," Am. Inst. Min. Met. Engrs., Tech. Pub. No. 2238, 1947.
(14) S. TIMOSHENKO, "Strength of Materials," Part 2, New York, Van Nostrand Book Co., 1956.
(15) J. MARIN, "Mechanical Properties of Materials and Design," New York, McGraw-Hill Book Co., 1942.
(16) C. R. SODERBERG, "Working Stresses," Chapter 10, Handbook for Experimental Stress Analysis, edited by M. Hetenyi, New York, John Wiley and Sons, Inc., p. 438, 1950.

# ANALYSIS OF ELECTRIC CIRCUITS CONTAINING NONLINEAR RESISTANCE

BY

LOUIS A. PIPES[1]

SYNOPSIS

This paper presents methods for the transient and steady-state analysis of electric circuits containing nonlinear resistances. It is shown how the "reversion method" for the solution of nonlinear differential equations may be applied to determine the transient response of circuits containing nonlinear resistors. A method for the determination of the steady-stage response of such circuits is also described. For purposes of illustration, the methods proposed are applied to the analysis of a circuit containing a linear inductor in series with a nonlinear resistor.

## I. INTRODUCTION

In recent years considerable attention has been directed to the analysis and to the study of the performance of electric circuits that contain nonlinear resistance elements or varistors (1).[2] These nonlinear resistance elements include the ceramic resistors called thyrites (2, 3, 4), silicon carbide resistors (5, 6) and certain types of colloidal resistors called polaristors (1, 7, 8).

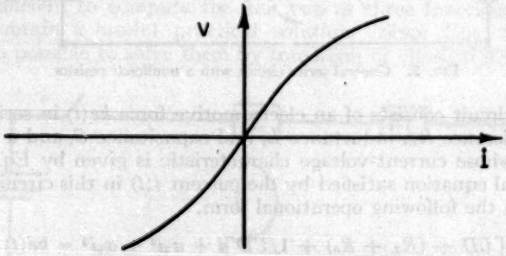



FIG. 1. Typical varistor characteristic.

These various nonlinear resistance elements have the typical current-potential characteristics shown in Fig. 1. For the purposes of mathematical analysis, the current-voltage characteristic curves of

[1] University of California and U. S. Naval Ordnance Test Station, Inyokern, China Lake, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

these elements may be described by the following empirical equation,

$$v(i) = R_0 i + a_3 i^3 + a_5 i^5 \tag{1}$$

where $i$ is the current flowing through the nonlinear element and $v(i)$ is the potential difference across its terminals. $R_0$ is given by the relation

$$R_0 = \frac{(dv)}{di} i = 0. \tag{2}$$

This is the *initial resistance* of the varistor. The coefficients $a_3$ and $a_5$ are determined by one of the standard methods of curve fitting (9).

The nonlinear resistors under consideration differ from other types of nonlinear resistors such as vacuum tube diodes, detectors, and rectifiers in that they are bilateral and exhibit a backward to forward ratio equal to unity.

### I. A GENERAL IMPEDANCE IN SERIES WITH A NONLINEAR RESISTOR

One of the principal problems in the study of circuits containing nonlinear resistances is the determination of the transient or the steady-state current of the general series circuit shown in Fig. 2.

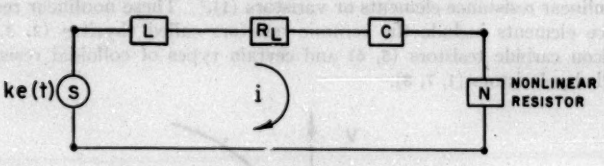



FIG. 2. General series circuit with a nonlinear resistor.

This circuit consists of an electromotive force $ke(t)$ in series with a linear resistance $R_L$, inductance $L$, and capacitance $C$, and a nonlinear element whose current-voltage characteristic is given by Eq. 1. The differential equation satisfied by the current $i(t)$ in this circuit may be written in the following operational form,

$$[LD + (R_L + R_0) + 1/CD]i + a_3 i^3 + a_5 i^5 = ke(t) \tag{3}$$

where $D$ is the operator,

$$D = \frac{d}{dt}. \tag{4}$$

Let the operator $a_1(D)$ be defined by the equation

$$a_1(D) = LD + (R + R_0) + \frac{1}{CD} \tag{5}$$

that is, $a_1(D)$ is the linear impedance operator of the circuit of Fig. 2. With this notation, Eq. 3 may be written in the following form,

$$a_1(D)i + a_3i^3 + a_5i^5 = ke(t). \tag{6}$$

This equation may be solved by the "Reversion Method" (10, 11, 12). To do this, assume a solution of Eq. 5 of the form

$$i = I_1(t)k + I_3(t)k^3 + I_5(t)k^5 + I_7(t)k^7 + \ldots \tag{7}$$

where the $I_i(t)$ quantities are functions of $t$. These functions satisfy a set of differential equations which are determined by substituting Eq. 7 into Eq. 6 and equating the coefficients of like powers of the resulting equation. If this is done, the following set of differential equations is obtained,

$$a_1(D)I_1(t) = e(t) \tag{8}$$

$$a_1(D)I_3(t) = -a_3I_1^3(t) \tag{9}$$

$$a_1(D)I_5(t) = -[3a_3(I_1^2I_3) + a_5I_1^5] \tag{10}$$

$$a_1(D)I_7(t) = -[3a_3(I_1^2I_5 + I_1I_3^2) + 5a_5(A_1^4A_3)]. \tag{11}$$

Additional equations may be found in (12). In most cases that arise in practice, $k = 1$ and the series (Eq. 7) converges rapidly. It is usually sufficient to compute the first two or three functions $I_i(t)$ in order to obtain a useful practical solution. Since Eqs. 8–11 are *linear*, it is possible to solve them by transform or classical methods.

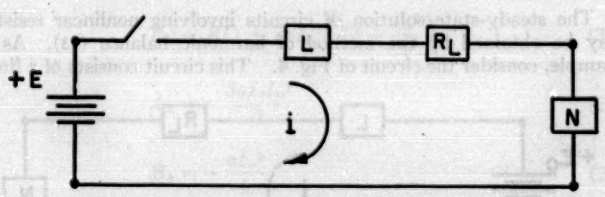



FIG. 3. Linear inductor and resistor in series with a varistor.

As a special case of the general theory, consider the circuit of Fig. 3. This circuit contains a linear resistor $R_L$ and inductor $L$ in series with a nonlinear resistor and a constant electromotive force $E$. Let it be required to determine the current $i(t)$ of the circuit under the assumption that at $t = 0$, $i = 0$. In this special case, the operator $a_1(D)$ has the form

$$a_1(D) = LD + R, \quad R = R_L + R_0. \tag{12}$$

The electromotive force $e(t) = E$ and hence $k = 1$ in Eq. 6. The solution for the current is, therefore,

$$i(t) = I_1(t) + I_3(t) + I_5(t) + \ldots \tag{13}$$

Since it is required that $i = 0$ at $t = 0$, the initial conditions for Eqs. 8–11 are that $I_3(t) = 0$ at $t = 0$.

In this special case Eq. 8 becomes

$$(LD + R)I_1(t) = E. \tag{14}$$

The solution of this equation with $I_1(0) = 0$ is

$$I_1(t) = \frac{E}{R}(1 - e^{-bt}), \qquad b = \frac{R}{L}. \tag{15}$$

The solution of Eq. 9 with $I_3(0) = 0$ is

$$I_3(t) = -a_3\frac{E^3}{R^4}[(1 - e^{-bt}) + 3(e^{-bt} - e^{-2bt}) - \tfrac{1}{2}(e^{-bt} - e^{-2bt}) - bte^{-bt}]. \tag{16}$$

The function $I_3(t)$ contains $R^6$ in the denominator and may be determined by solving Eq. 9. The series (Eq. 12) will in general converge. In practice the first two or three terms of Eq. 13 give practical information concerning the nature of the solution.

### III. STEADY-STATE SOLUTION OF CIRCUITS CONTAINING NONLINEAR RESISTORS

The steady-state solution of circuits involving nonlinear resistors may be obtained by the method of harmonic balance (13). As an example, consider the circuit of Fig. 4. This circuit consists of a linear

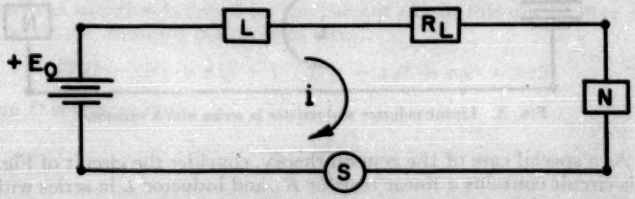



FIG. 4. Forced oscillations of a series circuit with a varistor.

inductor $L$ and resistor $R_L$ in series with a nonlinear resistor. The circuit is activated by a harmonic potential of the form $E_m \sin(wt + \theta)$

and a bias potential $E_0$. For simplicity, assume that the current-voltage characteristic of the nonlinear resistor is given by

$$v(i) = R_0 i + ai^3. \tag{17}$$

The differential equation satisfied by the current of the circuit is

$$\frac{Ldi}{dt} + Ri + ai^3 = E_0 + E_m \sin(wt + \theta), \quad R = R_L + R_0. \tag{18}$$

To determine the *steady-state* periodic solution of Eq. 18 assume an approximate solution of the form

$$i_0 = I_0 + I_m \sin(wt) \tag{19}$$

where $I_0$ is the direct current component of the circuit in the steady-state and $I_m$ is the amplitude of the fundamental of the steady-state alternating current of the circuit.

If $i_0$ is substituted into the left member of Eq. 18, the result may be written in the following form

$$F(t) = \frac{Ldi_0}{dt} + Ri_0 + ai_0^3 = A_0 + A_1 \cos(wt) + B_1 \sin(wt) + A_2 \cos(2wt) + B_3 \sin(3wt) \tag{20}$$

where

$$A_0 = RI_0 + aI_0\left(I_0^2 + \frac{3I_m^2}{2}\right) \tag{21}$$

$$A_1 = wLI_m \tag{22}$$

$$B_1 = RI_m + 3aI_m\left(I_0^2 + \frac{I_m^2}{4}\right) \tag{23}$$

$$A_2 = -\frac{3aI_0I_m^2}{2} \tag{24}$$

$$B_3 = -\frac{aI_m^3}{4}. \tag{25}$$

The right member of Eq. 18 may be written in the form

$$G(t) = E_0 + E_1 \cos(wt) + E_2 \sin(wt) \tag{26}$$

where

$$E_1 = E_m \sin(\theta) \tag{27}$$

$$E_2 = E_m \cos(\theta). \tag{28}$$

The principle of harmonic balance requires that the coefficients of the like harmonic terms of $F(t)$ and $G(t)$ should be equal if $i_0$ is to be an approximate solution of Eq. 18. If these coefficients are equated to each other, the following equations are obtained,

$$A_0 = E_0 \tag{29}$$

$$A_1 = E_1 \tag{30}$$

$$B_1 = E_2. \tag{31}$$

Let $N(t)$ be the error involved in the approximate solution $i_0$. The error is given by the equation

$$N(t) = G(t) - F(t) = -[A_2 \cos(2wt) + B_3 \sin(3wt)]. \tag{32}$$

The three equations (29–31) may be used to determine the unknown current amplitudes $I_0$ and $I_m$ and the phase angle $\theta$ of the approximate solution $i_0$. In terms of these amplitudes Eqs. 29–31 take the following form,

$$RI_0 + aI_0\left(I_0^2 + \frac{3I_m^2}{2}\right) = E_0 \tag{33}$$

$$wLI_m = E_m \sin(\theta) \tag{34}$$

$$RI_m + 3aI_m\left(I_0^2 + \frac{I_m^2}{4}\right) = E_m \cos(\theta). \tag{35}$$

In the special case in which the bias potential $E_0 = 0$, we have $I_0 = 0$ and Eqs. 34 and 35 reduce to

$$wLI_m = E_m \sin(\theta) \tag{36}$$

$$RI_m + \frac{3aI_m^3}{4} = E_m \cos(\theta). \tag{37}$$

If we let $y = I_m^2$ and add the squares of Eqs. 36 and 37, the result may be expressed in the form

$$\frac{9ay^3}{4} + \frac{3Ray^2}{2} + (w^2L^2 + R^2)y = E_m^2. \tag{38}$$

This is a cubic equation in $y$. If numerical values are given for the parameters involved, it may be solved graphically. The maximum value of the current $I_m$ is then given by

$$I_m = (y)^{1/2}. \tag{39}$$

Once $I_m$ is determined, the phase angle $\theta$ may be found from the equation

$$\sin(\theta) = \frac{wLI_m}{E_m}. \tag{40}$$

*The Case of Large Bias Potential $E_0 \gg E_m$*

An interesting special case that arises frequently in practice is the one in which the bias potential $E_0$ is much larger than the amplitude of the harmonic potential $E_m$, so that

$$E_0 \gg E_m. \tag{41}$$

In this case the direct component $I_0$ of the steady-state current, is much larger than the amplitude $I_m$ of the alternating component $I_0$, so that

$$I_0^2 \gg I_m^2. \tag{42}$$

In this case $I_m^2$ can be neglected in comparison with $I_0^2$ and Eq. 33 may be written in the form

$$RI_0 + aI_0^3 = E_0. \tag{43}$$

This cubic equation may be solved graphically for $I_0$. If the parameter $a$ is small, an approximate solution of Eq. 43 is

$$I_0 = \frac{E_0}{R}. \tag{44}$$

If the approximation (42) is made in Eq. 35, this equation may be expressed in the form

$$I_m(R + 3aI_0^2) = E_m \cos(\theta). \tag{45}$$

If Eqs. 34 and 45 are squared and the results added, the following equation for $I_m$ may be obtained,

$$I_m = E_m/[w^2L^2 + (R + 3aI_0^2)^2]^{1/2}. \tag{46}$$

The phase angle $\theta$ may be obtained by dividing Eq. 34 by Eq. 45. This operation gives the result

$$\tan(\theta) = wL/(R + 3aI_0^2). \tag{47}$$

The approximate steady-state current (Eq. 19) is, therefore,

$$i_0(t) = \frac{E_0}{R} + E_m \sin(wt)/(w^2L^2 + [R + 3a(E_0/R)^2]^2)^{1/2}. \tag{48}$$

If $a = 0$ the circuit reduces to a linear one and Eq. 48 is recognized as the usual steady-state current in the circuit. Equation 48 indicates that an increase in the bias potential produces an increase in the effective resistance of the circuit so that the amplitude of the alternating current can be controlled by varying the bias potential $E_0$.

### IV. DETERMINATION OF THE PRINCIPAL HARMONICS

In order to determine the amplitudes of the principal harmonics of the circuit, Eq. 18 may be written in the form

$$Ri = G(t) - \left[\frac{Ldi}{dt} + ai^3\right]. \tag{49}$$

Now if $i_0$ is substituted for $i$ in the right member of Eq. 49, the results can be expressed in the form

$$Ri = G(t) - F(t) + Ri_0 = N(t) + Ri_0 \tag{50}$$

where $F(t)$ is defined by Eq. 20 and $N(t)$ by Eq. 32.

Hence if $N(t)$ is expressed in terms of its harmonic components by Eq. 32, Eq. 50 may be written in the form

$$Ri = Ri_0 - [A_2 \cos(2wt) + B_3 \sin(3wt)]. \tag{51}$$

Therefore to this order of approximation the steady-state current of the circuit $i$ is given by the equation

$$i = I_0 + I_m \sin(wt) + \frac{aI_m^2}{2R}\left[3I_0 \cos(2wt) + \frac{I_m \sin(3wt)}{2}\right]. \tag{52}$$

This equation indicates that the current contains second and third harmonic components of the applied harmonic potential. Higher harmonic components may be computed if Eq. 52 is substituted in the right-hand member of Eq. 49. The convergence of the series obtained by repeating this procedure requires a separate investigation.

### V. CONCLUSION

In view of the numerous applications and practical importance of electric circuits involving nonlinear resistance elements in communication and control engineering, it is hoped that the methods of analysis of such circuits given in this paper will prove useful.

The number of analytical methods which have been developed for the study of nonlinear circuits is not very large and the applications of these techniques are usually laborious. However, when analytical procedures can be applied, a great deal of useful information concerning the performance of the circuit may be obtained.

REFERENCES

(1) H. E. HOLLMANN, "Nonlinear Elements and Some Applications in AF and RF Circuits," *Proc. Nat. Electronics Conference*, Vol. VII, pp. 130–140 (1951).
(2) K. B. MCEACHRON, "Thyrite, a New Material for Lightning Arresters," *G. E. Review*, Vol. 33, pp. 92–99 (1930).
(3) T. BROWNLEE, "Calculation of Circuits Containing Thyrite," *Ibid.*, Vol. 37, pp. 175–9 and 218–23 (1934).
(4) G. P. HARNWELL, "Principles of Electricity and Electromagnetism," New York, McGraw-Hill Book Co., Inc., 1938, pp. 141–145.
(5) T. ASHWORTH, A. NEEDHAM AND J. SILLARS, "Silicon-Carbide Non-Ohmic Resistors," *Proc. I.E.E.*, Vol. 93, Part I, p. 385 (1946).
(6) F. A. SCHWERTZ AND J. J. MAZENKO, "Nonlinear Semiconductor Resistors," *J. Appl. Phys.*, Vol. 24, pp. 1015–1024 (1953).
(7) H. E. HOLLMANN, "Semiconductive Colloidal Suspensions with Nonlinear Properties," *Ibid.*, Vol. 21, pp. 402–413 (1950).
(8) H. E. HOLLMANN, "Polaresistivity and Polaristors," *Proc. I.R.E.*, Vol. 40, pp. 538–545 (1952).
(9) I. S. SOKOLNIKOFF AND E. S. SOKOLNIKOFF, "Higher Mathematics for Engineers and Physicists," New York, McGraw-Hill Book Co., Inc., 1934, Chapt. XIV.
(10) L. A. PIPES, "The Reversion Method for Solving Nonlinear Differential Equations," *J. Appl. Phys.*, Vol. 23, pp. 202–207 (1952).
(11) A. C. SIM, "A Generalization of Reversion Formulae with their Application to Nonlinear Differential Equations," *Phil. Mag.*, Ser. 7, Vol. XLII, pp. 228–238 (1951).
(12) G. I. COHN, "Solution of Nonlinear Differential Equations by the Reversion Method," *J. Appl. Phys.*, Vol. 24, pp. 180–186 (1953).
(13) L. A. PIPES, "Forced Oscillations of Nonlinear Circuits," *Trans. A.I.E.E., Communications and Electronics*, Vol. 73, pp. 352–357 (1954).

**Automatic Jeep-Mounted Mine Detector.**—A jeep-mounted detector which automatically stops the vehicle when it locates a land mine has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Virginia. Capable of finding mines buried in and along roadways, the detector covers a path 6 ft. wide directly in front of the jeep and can be moved to both sides of the vehicle with ease. Operating controls, located on the dashboard of the jeep, can be manipulated by either the driver or his assistant.

The search coil, protected by an immersion proof rubber-coated plywood box, is kept at a constant height above the ground by three skid-caster type wheels which support its fiberglass carriage assembly.

When the search coil passes over a mine, the vehicle is stopped instantly by the brake actuator, a spring-loaded hydraulically operated piston which disengages the clutch and sets the brakes.

A meter, mounted on the indicator panel indicates the presence of the mine. A system of lights, also located on the panel, changes from green to red and an audible signal is received through earphones by the driver.

The detector and its carriage assembly weigh about 700 lb. When not in operation, the apparatus is transported in carrying racks which are provided on the rear and on one side of the jeep.

**Gasifying Coal in the Seam.**—A promising method of gasifying coal in the original seam underground was described in a British paper presented to the Joint Solid Fuels Conference in Washington. Preliminary estimates indicate that gas may be brought to the surface at half the cost of mined coal of equal heating value. Much remains to be done to confirm these early findings and the necessary work is being pushed forward by the National Coal Board.

The gas that reaches the surface can be burnt to raise steam for generating electricity. The development of the necessary equipment is the responsibility of the Central Electricity Authority.

It is hoped that a pilot plant generating 5000 kW will be in operation in 1959, to be followed by a full scale installation of 60,000 kW capacity by 1961.

The method has been worked out by the Ministry of Fuel and Power to make use of at least 300 million tons of coal in Britain which is in seams too thin to be mined in the usual way and it will be further developed to enable this otherwise inaccessible fuel to contribute to the country's overtaxed energy resources.

Shafts are to be sunk from the surface to the level of the coal and parallel galleries driven through the coal at about 500-yd. intervals. From each gallery a large number of holes will be bored at right angles into the coal for a distance of about 200 yards. These boreholes will be blind ended and will each be equipped with an internal pipe carrying air to keep the coal burning after it has been lit electrically at the far end. The inflammable gas will then pass to the surface back through the borehole along the galleries and up the shaft to the steam raising plant on the surface.

In experiments using four boreholes 1700 tons of coal was gasified during 4 months of continuous operation after the seam was lit in August, 1955. Another smaller experiment showed that the coal around a single borehole could be kept alight for 7 months during which about 300 tons of coal were burnt. These were among the recent hopeful results of work which has been going on since late 1952.

## NOTES FROM THE NATIONAL BUREAU OF STANDARDS

### CONTROLLED BURNING OF COMBUSTIBLE MATERIALS

The National Bureau of Standards has developed a simple, rapid method for burning combustible materials under closely controlled conditions. With this procedure and equipment, the gaseous products formed when organic materials are burned in air can readily be collected and analyzed.[1] The results give a quantitative estimate of the combustion gases produced from such organic coatings as paints, asphalts, and plastic compounds. This information is useful in selecting organic coatings with particular thermal breakdown properties for use in buildings and other structures. The method was developed by A. Schriesheim, of the Bureau's floor, roof, and wall coverings laboratories, working under the sponsorship of the U. S. Air Force.

The principal combustion products of organic materials are carbon monoxide, carbon dioxide, and water. When organic materials contain other elements in addition to carbon, hydrogen, and oxygen, combustion yields other gases as well. Among these are ammonia from wool, cyanogen from silk, sulfur compounds from rubber, and chlorine compounds from chlorinated plastics. To generate, collect, and analyze combustion products, earlier investigators burned large, built-up specimens, including actual rooms and buildings. Subsequently, laboratory procedures were developed to replace such costly and time-consuming processes, but these methods were slow and of doubtful accuracy. The Bureau's recently developed method of burning organic materials provides a quick and comprehensive analysis of the gases as well as close control of the ratio between air volume and specimen weight at any initial firing temperature up to 550° C.

The Bureau's equipment for burning organic materials consists essentially of a combustion chamber containing a heating element for firing the specimen, and apparatus to control the amount of air in the chamber. The combustion chamber, a two-liter pyrex flask, can be tilted so that the specimen will slide down a silica tube inside the flask into the heating element.

In use, a porcelain boat containing the specimen is placed in the silica tube as far from the platinum heating coil as possible. After the combustion chamber is evacuated through the vacuum system, a controlled amount of air is admitted to the chamber. A constant ratio

[1] For further technical details, see "Method for the Controlled Burning of Combustible Materials and Analyses of the Combustion Gases," by Alan Schriesheim, *J. Research NBS*, Vol. 57, p. 245 (1956) RP 2715.

of specimen weight to amount of air is maintained by adjusting the specimen weight when combustion is initiated at higher temperatures.

Current is applied to the platinum coil until the desired firing temperature is reached. The combustion chamber is inclined, the boat containing the specimen slides down the silica tube into the hot platinum coil, and the material begins burning. After the temperature has been held constant for the appropriate time, the current is turned off and the apparatus cools to room temperature. The combustion gases are now available for analysis in a mass spectrometer.

In a series of investigations performed with this equipment, the results showed that for every material examined the greatest variety of gases was produced at the highest initial firing temperature (550° C.). At this temperature, cracking and decomposition occurred and small molecular fragments such as methane and hydrogen were formed from many of the specimens. Chlorinated plastics liberated chlorinated compounds at the higher temperatures, while at the lowest firing temperature the only chlorinated material produced was hydrogen chloride. In every case the amount of hydrogen chloride increased with increasing temperature.

Several plywood assemblies, both painted and unpainted, and a polyvinyl chloride coating were fired in quantities sufficient to consume all the oxygen in the combustion chamber if they had been left to burn entirely to carbon dioxide and water. These materials produced the largest concentration of combustion gases, and caused the greatest decrease in the oxygen concentration. This low oxygen concentration is typical not only of the closed system used here but also of large, open, ventilated systems such as burning rooms and buildings.

## HIGH-PURITY BARIUM TITANATE

Barium titanate is now being used extensively in electronic devices because of its desirable dielectric and piezoelectric properties. Important applications of this ceramic include high-capacity miniature condensers, ultrasonic transmitters, and transducers for sensing devices in accelerometers, sound detectors, velocimeters, and strain and pressure gages. However, in spite of its wide use, the relationship between composition and electrical characteristics of barium titanate has not been fully worked out.

To aid in the solution of this problem, the National Bureau of Standards recently developed a method[1] for preparing very-high-purity barium titanate in which the mole ratio of barium to titanium is exactly unity. Using the pure material, ceramic engineers should now be able to determine systematically the effect of various minor components on

[1] For further technical details, see "Preparation of Barium Titanyl Oxalate Tetrahydrate for Conversion to Barium Titanate of High Purity," by W. Stanley Clabaugh, Edward M. Swiggard, and Raleigh Gilchrist, *J. Research NBS*, Vol. 56, p. 289 (1956) RP 2677.

the electrical properties of barium titanate and to arrive at a reproducible composition having the most desirable electrical characteristics.

The procedure for preparation was devised by W. S. Clabaugh, E. M. Swiggard, and Raleigh Gilchrist of the Bureau's inorganic chemistry laboratory, working under the sponsorship of the Diamond Ordnance Fuze Laboratories, Department of the Army. The method involves, first, preparation of the double oxalate tetrahydrate, $BaTiO(C_2O_4)_2 \cdot 4H_2O$. This compound is then ignited, yielding a pure barium titanate containing equal molar proportions of barium and titanium.

The oxalate is prepared by very slowly adding an aqueous solution of titanium tetrachloride and barium chloride to a hot solution of oxalic acid. The titanium tetrachloride and barium chloride are in nearly equimolar proportion but with barium in slight excess, while the amount of oxalic acid is about 10 per cent more than the reaction requires. The purpose of the excess barium and oxalate is to minimize competing reactions that might result in formation of insoluble partially hydrolyzed compounds of titanium.

While the solutions are being added, the oxalic acid solution is continuously heated at 80° C. Eventually a dense crystalline precipitate of $BaTiO(C_2O_4)_2 \cdot 4H_2O$ is formed. This substance can then be converted to barium titanate by heating in a furnace at 900° C.

The titanium tetrachloride solution is made by adding the highly purified tetrachloride, drop by drop, to rapidly stirred distilled water cooled in an ice bath. This procedure prevents formation of hydrolyzed titanium compounds. The exact titanium content of the solution must then be determined so that the quantities of barium chloride and oxalic acid to be used can be based on this value.

# THE FRANKLIN INSTITUTE

## MINUTES OF THE STATED MEETING

**December 19, 1956**

The Stated Monthly Meeting of The Franklin Institute was held December 19, 1956, in the Lecture Hall. Mr. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. with approximately 325 members and guests in attendance.

The President announced that the minutes of the November Stated Meeting had been printed in the December JOURNAL OF THE FRANKLIN INSTITUTE; there being no corrections, changes or additions, they were approved as published.

Cecil M. Waterbury, Assistant Secretary, reported that the following nominations for election to the Board of Managers had been made in conformity with Article IV, Section 5, of the By-Laws:

| | |
|---|---|
| Brandon Barringer | Charles S. Redding |
| Edward G. Budd, Jr. | John Russell, Jr. |
| James McGowan, Jr. | W. Maxwell Scott, Jr. |
| Richard T. Nalle | I. Melville Stein |

The President then called for nominations from the floor. There being none, nominations were declared closed. George S. Crampton, Joseph S. Hepburn and Howard Stoertz, who are neither officers nor nominees, were appointed by the President to act as tellers of the election.

After announcing that this was the first of the Charles S. Redding Lectures since the Lectureship was established in 1955 by the Leeds and Northrup Foundation, Mr. Rolph introduced Mr. and Mrs. Redding.

The President then introduced Dr. Gaylord P. Harnwell, whose talk on "The Atomic Nucleus and Modern Technology" was very interesting.

The meeting adjourned at 9:35 P.M.

CECIL M. WATERBURY
*Assistant Secretary*

## MEMBERSHIP

### ACTIVE MEMBERS ELECTED BY THE BOARD OF MANAGERS

December 19, 1956

#### ACTIVE FAMILY

Victor F. Hanson
Shelby L. Jones
Edward L. Morin
Herbert Trotter, Jr.

#### ACTIVE

James W. Althouse, Jr.
R. H. Bickford
James J. Crawford
R. Jacques T. duPont
John K. Durkin
Paul M. Fisher
Norman P. Gentieu
Alvin D. Gottlieb
James Hillier
John J. Keane, Jr.
Adam Kruczay, Jr.
Walter H. Lenhard, Jr.
Mrs. James McMullan
L. Parker Naudain
Stig B. Poulsen
C. A. Rumpff
Robert Gwynne Stout

#### ACTIVE NON-RESIDENT

M. K. Blanchard
Allan B. Crunden

#### NECROLOGY

Frank H. Bachman '55
A. H. Manwaring, 2nd '48
Rodney K. Merrick '54
Irvin Michaelson '54
George F. Nelson '47
Henry N. Platt '36
Clarence E. Rose '54
Harry Rubin '47
F. Carter Schaub '48

## LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 231 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

### RECENT ADDITIONS

#### AERONAUTICS

LIPTROT, ROGER NORMAN AND WOODS, JAMES DENIS. Rotocraft. 1955.

SUTTON, GEORGE P. Rocket Propulsion Elements; an Introduction to the Engineering of Rockets. Ed. 2. 1956.

#### BIOGRAPHY

TRAVERS, MORRIS W. A Life of Sir William Ramsay. 1956.

#### CHEMISTRY AND CHEMICAL TECHNOLOGY

BILLITER, JEAN. Die Technische Elektrolyse der Nichtmetalle. 1954.

BOSWORTH, RICHARD CHARLES LESLIE. Transport Processes in Applied Chemistry. 1956.

REMY, GERHARD EBERHARD HEINRICH. Grundriss der Anorganischen Chemie. Ed. 6. 1956.

TODD, SIR ALEXANDER, ED. Perspectives in Organic Chemistry. 1956.

TRUTER, E. VERNON. Wool Wax; Chemistry and Technology. 1956.

VAN ALPHEN, J. Rubber Chemicals. 1956.

WEEKS, MARY ELVIRA. Discovery of the Elements. Ed. 6. 1956.

### ELECTRICITY AND ELECTRICAL ENGINEERING

BECKER, ARTHUR AND VOIGT, HANSKARL. Mathematisches Hilfsbuch für die Wechselstromtechnik. Ed. 5. 1955.
BRECKENRIDGE, R. G.; RUSSELL, B. R. AND HAHN, E. E., ED. Photoconductivity Conference. 1956.
FEDERAL TELEPHONE AND RADIO CORPORATION. Reference Data for Radio Engineers. Ed. 4. 1956.
FLECK, BOTHO. Hochspannungs- und Nieder-Spannungs-Schaltanlagen. Ed. 3. 1955.
FOWLER, CHARLES. High Fidelity. 1956.
MANDL, MATTHEW. Handbook of Basic Circuits: TV, FM, AM. 1956.
POSTNIKOW, I. M. Die Wahl Optimaler Geometrischer Abmessungen Elektrischer Maschinen. 1955.
STEWART, JOHN L. Circuit Theory and Design. 1956.
TIBBS, CHRISTOPHER E. AND JOHNSTONE, G. G. Frequency Modulation Engineering. Ed. 2. 1956.
VAN BLADEL, J. Les Applications du Radar à l'Astronomie et à la Météorologie. 1955.
ZBAR, PAUL B. Basic Electricity. 1956.
ZBAR, PAUL B. AND SCHILDKRAUT, SID. Basic Electronics. 1956.
ZBAR, PAUL B. AND SCHILDKRAUT, SID. Basic Radio and Radio-Receiver Servicing. 1956.

### ENGINEERING

BATRAKOW, W. P. Korrosion Metallischer Werkstoffe in Aggressivenmitteln. 1954.
CRANDALL, STEPHEN H. Engineering Analysis. 1956.
KEYSER, CARL A. Materials of Engineering. 1956.

### FOOD

SCHWITZER, M. K. Margarine and Other Food Fats. 1956.

### GENERAL

FIRTH, MARGARET A., ED. Handbook of Scientific and Technical Awards in the United States and Canada, 1900–1952. 1956.
KELLOGG W. K. FOUNDATION. The First Twenty-five Years. n.d.

### GEOLOGY

NININGER, ROBERT D., ED. Exploration for Nuclear Raw Materials. 1956.

### HOROLOGY

YDE-ANDERSEN, DAVID. Bornholmere. 1953.

### INDUSTRIAL MANAGEMENT

MAYNARD, HAROLD BRIGHT, ED. Industrial Engineering Handbook. 1956.

### MANUFACTURE

COOK, PHILIP G. Latex. 1956.
HOLLAND, L. Vacuum Deposition of Thin Films. 1956.

### MATHEMATICS

BAUER, EDMOND. Champs de Vecteurs et de Tenseurs. 1955.
BEHNKE, HEINRICH AND SOMMER, FRIEDRICH. Theorie der Analytischen Funktionen einer Komplexen Veränderlichen. 1955.
BODEWIG, E. Matrix Calculus. 1956.
JOHNSON, CLARENCE L. Analog Computer Techniques. 1956.
KNOPP, KONRAD. Infinite Sequences and Series. 1956.
MEIXNER, JOSEF AND SCHÄFKE, FRIEDRICH WILHELM. Mathieusche Funktionen und Sphäroidfunktionen. 1954.
NEWSKI, B. A. Praktikum der Nomogrammkonstruktionen. 1955.
NIVEN, IVAN. Irrational Numbers. 1956.

### MECHANICAL ENGINEERING

BOBEK, KARL; HEISS, A. AND SCHMIDT, FR. Stahlleichtbau von Maschinen. Ed. 2. 1955.
CARROLL, JOHN M. Mechanical Design for Electronics Production. 1956.
GREEN, WALTER GEORGE. Theory of Machines. 1955.

### METALLURGY

AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS. INSTITUTE OF METALS DIVISION. Nuclear Metallurgy. No. 1–3. 1956.
HOUDREMONT, EDUARD. Handbuch der Sonderstahlkunde. Ed. 3. Vol. 1 and 2. 1956.
KUBASCHEWSKI, OSWALD AND CATTERALL, J. A. Thermochemical Data of Alloys. 1956.
OHMANN, HELMUT. Allgemeine und Praktische Metallkunde. 1955.
UDY, MARVIN J. Chromium. Vol. 2. 1956.

### NUCLEAR ENGINEERING

CHARPIE, R. A. *et al.*, ED. Reactors. 1956.

### PHYSICS

ARZELIÉS, HENRI. La Cinématique Relativiste. 1955.
CARMAN, P. C. Flow of Gases Through Porous Media. 1956.
COMINGS, EDWARD W. High Pressure Technology. 1956.
INTERNATIONAL CONFERENCE OF THEORETICAL PHYSICS. Proceedings. 1954.
JONES, GWYN OWAIN *et al.* Atoms and the Universe. 1956.
KITTEL, CHARLES. Introduction to Solid State Physics. Ed. 2. 1956.
KÖSTER, W. Beiträge zur Theorie des Ferromagnetismus. 1956.
KOUDRIAVTSEV, V. Le Monde des Ultrasons. 1955.
LIECHTI, ADOLPH. Röntgenphysik. Ed. 2. 1955.
LUDWIG, GÜNTHER. Die Grundlagen der Quantenmechanik. 1954.
McVITTIE, G. C. General Relativity and Cosmology. 1956.
SEARS, FRANCIS WESTON AND ZEMANSKY, MARK W. University Physics. Ed. 2. 1955.
TEMPERLEY, H. N. V. Changes of State. 1956.

### TEXTILES

LINDE, W. W. AND OSSIPOW, P. A. Die Seide. 1954.
POLANYI, M., COMP. Dictionary of Textile Terms. 1956.

## JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

LAND, EDWIN H.: From Imbibition to Exhibition
HILL, E. L.: Electromagnetic Radiation from Lightning Strokes
SLEPIAN, JOSEPH: The Magneto-Ionic Expander Isotope Separator
WEIDLEIN, EDWARD R.: Organizing for Effective Air Pollution Control
COULSON, THOMAS: Some Prominent Members of The Franklin Institute. 4. Coleman Sellers, 1827–1907
PIPES, LOUIS A.: Matrix Analysis of Heat Transfer Problems
BOLINDER, E. FOLKE: The Classical Electromagnetic Equations Expressed as Complex Four-Dimensional Quantities
MADAN, MAHENDRA P.: Simple Bridge Method for the Measurement of Thermal Conductivity of Gases and Gas Mixtures

**New Alloy for Aircraft.**—The Dow Chemical Company has announced the second in a series of high temperature magnesium-thorium alloys. Both the new alloy—HM21XA-T8—and its predecessor—HK31A—are airframe materials designed to build higher performance supersonic aircraft and missiles, the company said. Officials said enough of the new alloy is on hand for immediate evaluation in the supersonic craft.

The company declared both the new alloys were designed to meet a critical defense need: A structural material for aircraft and missiles that combines light weight and strength with resistance to heat generated at supersonic speeds. Key to the success of alloys is magnesium's favorable combination of light weight and strength. Other materials bear up well under elevated temperatures, spokesmen said, but their greater weight holds down maximum performance.

Dow engineers say HM21XA-T8, the newer material, is designed to withstand up to 100 hours of exposure at temperatures of 700° F. with little effect on its properties. HK31A retains good short-time properties above 600° F. and good long-time properties below that temperature.

Dow engineers report HM21XA-T8 shows improved resistance to creep and otherwise retains many of the characteristics of the older material. Magnesium's high strength-weight ratio means both alloys can be used in thicker structural sections, making them far more resistant to buckling than other materials, they added.

**TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE**

**All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.**

**There is still a continuing and pressing need of contributions and bequests —both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.**

**Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.**

**Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.**

**We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.**

# NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

## TYPEWRITERS

The typewriter plays such an important part in both business activities and personal correspondence of the whole world that there is no need to enlarge upon its influence. We are also so familiar with the part played by women in commercial life that we might overlook the role it played in their so-called "emancipation."

It was not until the invention of printing in the middle of the fifteenth century that we were able to free ourselves from slavery to the pen; before that time every document and book had to be written by hand. Typewriting is an intermediate step between handwriting and printing. Today, all writing by hand except the signing of names has disappeared from many businesses.

The typewriter as we know it is only 83 years old, for we count its beginning from 1873 when the Sholes and Glidden machine was placed upon the market. Many inventors during the preceding half century had produced experimental machines which appear comparable in merit with that which Sholes and his associates had to offer, but the times were not ripe for the invention. It is worth noting, too, that many of the earliest efforts were inspired by the desire to furnish the blind with the means of reading and writing. Here speed was of little account, and it was not until the machine became capable of surpassing the pen for speed that its potentialities were appreciated. Until it was realized that shorthand and its transcription on the typewriter freed a busy man from the drudgery of the pen, the typewriter remained an interesting novelty.

When the typewriter made its first serious appeal to the public at the Centennial Exposition of 1876 it attracted very little attention. Groups of salesmen were discouraged in their efforts to advance sales. They had no other aid than a neat little catalogue which prophetically called the typewriter "A Machine to Supersede the Pen."

Like all other machines the component parts have undergone considerable modification with the passage of the years. Around some were more or less violent controversies, as for instance whether direct inking was preferable to ribbon inking. The former was claimed to give a clearer impression, but it never achieved popularity because the self inking apparatus required constant attention. Another controversy raged over the double keyboard as opposed to the single keyboard with a shift. The spread of the touch system of operation

decided in favor of the universal adoption of the single keyboard. Another point was the value of the differential spacing of the characters, but in America this has been dismissed as an unnecessary requirement, except in the more recent examples of some electric typewriters, which can justify a line of type.

An exhibit in the Museum furnishes a bird's eye view of the typewriter's development, showing the different fundamental characteristics. The swinging sector group has the type characters on a rotating cylinder or segment. Another group is represented by the radial strike design, where the stroke is made from a position approximating to a right angle to the platen, frequently employed in the noiseless types. Type bars have been made to strike upward or downward. In those which struck upward the writing was invisible until the platen was turned over by hand. These old types of machine are very rarely seen in use today. The common typewriter of today strikes from in front and the writing is always in sight.

The electric typewriter is the aristocrat of machines. The mechanical effort to operate it is only a fraction of that required on the conventional manually operated machines. The heavier operations, such as the carriage shift, change for capitals, etc., are equally facilitated. The work produced on these machines can easily be recognized by its clean and uniform impression.

The exhibit, furnished by International Business Machines, is located in the Graphic Arts Section.

THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

# VIBRATION ANALYSIS OF RAILROAD TRACK

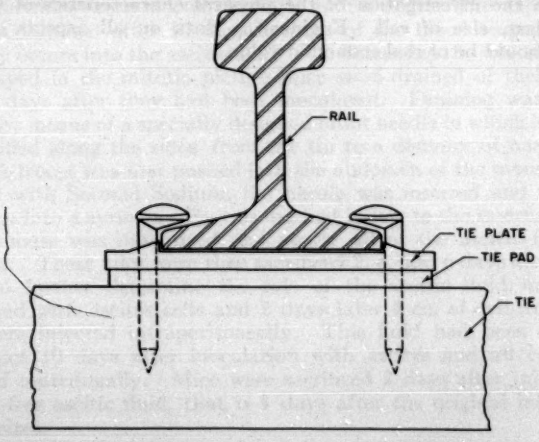
BY

AXEL O. BERGHOLM

In recent years the railroads in this country have been placing a fabric pad between the tie plates and the ties in their track systems. These pads were initially installed to keep moisture away from that underplate portion of the tie surface where it can cause progressive destruction of the tie. A properly designed and constructed tie pad will be so securely stuck to the tie that it will become integral with it. Moisture and abrasive materials cannot penetrate the seal between the tie pad and the tie. It has also been reported that these pads reduce rail joint wear and roadbed raveling. Under the sponsorship of Bird and Son, Inc. the Engineering Mechanics Section investigated the actual motion of rail and ties in the field and determined to what extent various pads affect track and tie vibration.

This work consisted of two vibration surveys conducted on the New York, New Haven, Hartford Railroad at Old Lyme and at Westbrook, Conn. These surveys were performed on padded and unpadded tracks. Comparative vibration data were obtained for several different types of pads.

The instrumentation consisted of vibration pickups attached to the bottom flange of the rails and to the ties. Displacement pickups



measured the motion between the rail and tie, rail and roadbed, and between the tie and roadbed. The output of these instruments was recorded by an oscillograph.

Before the experimental work was planned it was reasoned that there are several ways in which the pads may change the roadbed vibrations and thus produce beneficial results. First, the pads may act as an additional spring in the roadbed and thus produce a new vibratory system with different frequencies. Second, the pads may increase the amount of damping in the track system and thus reduce the vibratory motion and damage to the roadbed. Third, the reduction of rail vibration may contribute to a more stable ballast. Fourth, the ties may remain tighter and thus provide a more stable track structure.

A study of the test results obtained on the padded and unpadded rails revealed differences in the vibration characteristics which can be attributed to the pads. Some of these are: (1) the wave front in the rails ahead of a train is reduced; (2) the pulses generated as the front wheels of a locomotive hit the rail joint preceding the joint of test rail are eliminated; (3) the magnitude of the rail vibration generated by the front wheels of the locomotive as they hit the leading rail joint is reduced; (4) the tie vibration during the period when the locomotive is directly over the rail is reduced; (5) vibrations are damped away in a fewer number of cycles.

No doubt the magnitude of differences depended on the physical characteristics of the tie pads. No attempt was made in this study to determine the relation between the tie pads and the differences in the vibration that were observed. Enough work has been done to stimulate interest in the investigation of the physical characteristics of the tie pads, ballast, size of rail. Engineering data on all aspects of this problem should be of real economic value.

THE BIOCHEMICAL RESEARCH FOUNDATION
ELLICE McDONALD, FOUNDER
WILLIAM G. BATT, DIRECTOR

# THE MITOTIC PICTURE IN A GROWING POPULATION OF KREBS 2 ASCITES TUMOR CELLS

BY

IRVING GALINSKY AND WILLIAM G. BATT

The mitotic picture of the Krebs 2 ascites tumor has been studied from the time of inoculation into mice until the death of the animals. There had been comparatively little information concerning the continuous mitotic aspect of the tumor; the studies of mitosis in ascites which were reported concerned themselves mainly with chromosome numbers (1, 2), morphology (3) and chemical treatment (4, 5). In order to obtain a clearer picture, an analysis was made of the different mitotic stages, both normal and abnormal, present in the Krebs 2 ascites tumor cells at various intervals after their inoculation into the mouse.

## MATERIALS AND METHODS

The Krebs 2 ascites tumor used has been maintained in this laboratory in Swiss female mice. The original tumor was obtained from Dr. T. S. Hauschka in 1952. Swiss female mice were inoculated, intraperitoneally, with approximately $5 \times 10^6$ cells obtained from a mouse which had been inoculated 8 days previously. The mice were sacrificed by cervical fracture.

An analysis of the different mitotic stages, both normal and abnormal, present in the Krebs 2 ascites tumor was made 2, 4, 6, 8 and 10 days after mice were inoculated with ascites cells. The experiments were discontinued after 10 days because of the blood seepage which generally occurs into the ascitic fluid. To determine the role the ascitic fluid played in the mitotic picture mice were drained of their ascitic fluid, 8 days after they had been inoculated. Draining was accomplished by means of a specially designed blunt needle in which holes had been drilled along the sides, from the tip to a distance of one-quarter inch. A trocar was first pushed into the abdomen of the mouse anaesthetized with Seconal Sodium, the needle was inserted and the fluid drawn up into a syringe. By this method injury to the internal organs of the mouse was minimized and nearly all of the ascitic fluid was removed. These mice were then sacrificed 2, 4, and 6 days after draining. To further determine the role of the ascitic fluid mice were inoculated with ascites cells and 2 days later 4 cc. of cell-free ascitic fluid were injected intraperitoneally. This fluid had been obtained from mice 10 days after inoculation with ascites and all cells were removed centrifugally. Mice were sacrificed 2 days after injection of the cell-free ascitic fluid, that is 4 days after the original inoculation with ascites.

Slides were made of the ascites cells by the smear technique and aceto-orcein staining as described by Tjio and Levan (6). The various stages of mitosis, both normal and abnormal, were scored by examining 1000 mitotic cells. For each mouse, the percentage of abnormal interphase cells was calculated on 1000 cells, resting and mitotic. The mitotic index was also determined on the basis of 1000 cells, resting and mitotic. The results from four mice were averaged for each figure shown in the tables and the standard error of the mean was calculated.

## RESULTS

The percentage of abnormal mitoses remained low until about 6 days after inoculation as shown in Table I. However, from the eighth

TABLE I.—*Percentage* of Normal and Abnormal Mitoses in Krebs 2 Ascites Cells and the Influence of Ascitic Fluid on this Proportion.*

| | Days after inoculation | Mitoses | |
|---|---|---|---|
| | | Normal | Abnormal |
| | 2 | 87.8 | 12.2 |
| | 4 | 85.2 | 14.8 |
| | 6 | 81.6 | 18.4 |
| | 8 | 64.5 | 35.5 |
| | 10 | 58.0 | 42.0 |
| Ascitic fluid drained 8 days after inoculation | 10 | 73.6 | 26.4 |
| | 12 | 55.1 | 44.9 |
| | 14 | 47.3 | 52.7 |
| Ascitic fluid (cell-free) injected 2 days after inoculation | 4 | 81.0 | 19.0 |

* Each calculation based on 1000 *mitotic* cells.

day on there was a decided increase in the percentage of abnormal mitoses. The different stages of mitosis of the ascites cell population after inoculation into mice are shown in Table II. The proportion of prophases was decidedly decreased until at 10 days it was almost half what it had been at 2 days. There was a steady, but not as striking an increase in the proportion of metaphases, while that of anaphases and telophases remained fairly constant. The range of mitotic abnormalities was similar to that reported for other tumors and also for cells treated with mitotic poisons. The greatest proportion of abnormalities was found in the metaphase stage.

The most frequent metaphase abnormality began with the chromosomes arranged on the equatorial plate (Fig. 1). The centromeres

TABLE II.—*Percentage* of Different Mitotic Stages in Krebs 2 Ascites Cells and the Influence of Ascitic Fluid on this Frequency.*

| | Days after inoculation | Prophase | Metaphase | | | Anaphase | | | Telophase |
|---|---|---|---|---|---|---|---|---|---|
| | | | Normal | Abnormal | Total | Normal | Abnormal | Total | |
| | 2 | 40.1±1.26** | 30.7±2.00 | 9.8±1.50 | 40.5±0.99 | 7.9±1.19 | 2.4±0.78 | 10.3±1.21 | 9.2±1.33 |
| | 4 | 34.7±2.34 | 29.2±1.55 | 12.6±1.95 | 41.8±1.91 | 9.7±2.40 | 2.2±0.42 | 11.9±2.30 | 11.7±0.83 |
| | 6 | 33.0±2.26 | 30.8±2.54 | 15.2±3.16 | 46.0±0.73 | 8.6±1.91 | 3.2±0.86 | 11.8±2.19 | 9.3±1.67 |
| | 8 | 28.6±2.88 | 21.9±1.61 | 31.5±4.18 | 53.4±2.90 | 5.6±0.26 | 4.0±0.61 | 9.7±0.72 | 8.2±0.54 |
| | 10 | 24.2±1.14 | 18.5±4.02 | 39.2±1.85 | 57.6±3.27 | 5.5±1.36 | 2.8±1.58 | 8.3±2.53 | 9.9±1.55 |
| Ascitic fluid drained 8 days after inoculation | 10 | 25.3±1.89 | 28.8±0.77 | 24.6±2.03 | 53.5±1.83 | 7.8±1.89 | 1.8±0.50 | 9.6±1.93 | 11.6±2.15 |
| | 12 | 16.3±4.77 | 14.6±2.29 | 42.2±2.97 | 56.8±3.42 | 9.1±1.88 | 2.7±0.93 | 11.8±1.91 | 15.3±2.88 |
| | 14 | 21.2±4.01 | 7.6±1.79 | 48.6±2.22 | 56.2±3.90 | 3.9±0.08 | 4.1±1.25 | 8.0±1.27 | 12.9±1.51 |
| Ascitic fluid (cell-free) injected 2 days after inoculation | 4 | 23.8±1.06 | 30.1±2.60 | 16.9±3.10 | 47.0±0.55 | 11.0±0.60 | 2.1±0.12 | 13.1±0.61 | 16.1±0.28 |

* Each calculation based on 1000 *mitotic* cells.
** Standard error of the mean.

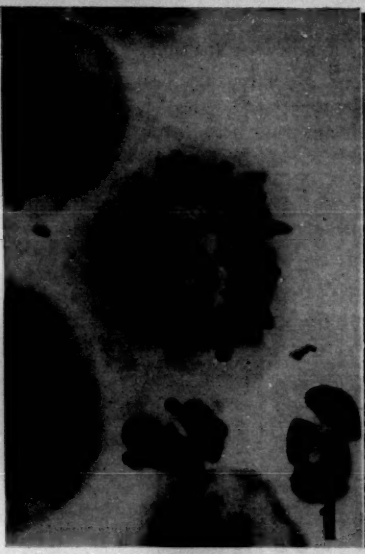


FIG. 1. Normal metaphase. × 1900.

FIG. 2. Abnormal metaphase forming interphase nucleus, without any anaphase occurring. × 1900.

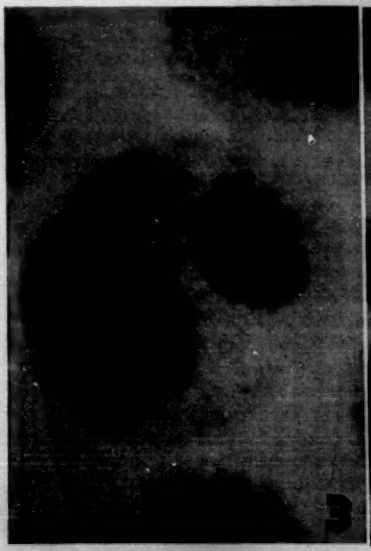


**FIG. 3.** Abnormal multinucleate interphase cell. × 1900.

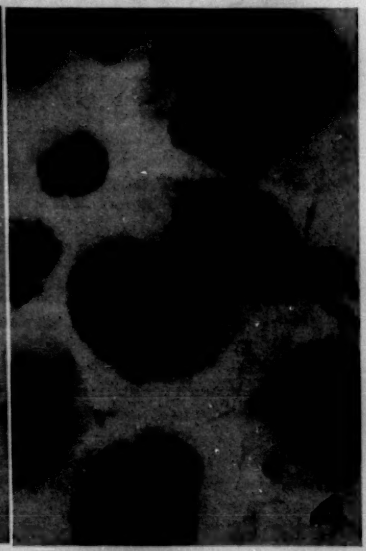


FIG. 4. Abnormal interphase cell with dumbbell-shaped nucleus. × 1900.

failed to split and the spindle did not function. Consequently, the metaphase chromosomes went directly into the telophase stage without any intervening anaphase stage. The chromosomes gradually lost their distinct outlines and became diffuse in appearance. In a number of cases the nucleic acid seemed to spread out over the chromosomes giving them a clumped appearance. As the nucleic acid continued to disappear the chromosomes took on a netlike appearance and formed the chromatin network of the interphase nucleus. The centromere was rarely found to have divided before the chromosomes had formed into a restitution nucleus. Lobed or hollow nuclei were sometimes formed depending upon the arrangement of the chromosomes on the equatorial plate (Fig. 2). The reconstituted nuclear membrane appeared to follow the outline, however irregular, of the chromosome group.

The frequency of endomitosis in the Krebs 2 ascites was so low that these counts were not included in the tables. The infrequency of endomitosis had been observed by Levan (7) in the Krebs 2 ascites.

The anaphase variations were almost all of the lagging chromosome or bridge chromosome type. Some tripolar and tetrapolar anaphases, however, also occurred. Failure of the cell to divide after the chromosomes reached the poles resulted in binucleate, trinucleate and tetranucleate cells and sometimes cells with dumbbell-shaped nuclei. If the cells contained more than one nucleus or if the nucleus was dumbbell-shaped (Figs. 3 and 4) they were classified as abnormal interphase. The majority of the abnormal interphase cells shown in Table III were binucleate cells. There was a decrease in the number of abnormal

TABLE III.—*Percentage* of Abnormal Interphase Cells in Krebs 2 Ascites Tumor and the Influence of Ascitic Fluid on the Frequency.*

| | Days after inoculation | Abnormal interphase cells |
|---|---|---|
| | 2 | 1.2 ± 0.46** |
| | 4 | 1.7 ± 0.64 |
| | 6 | 2.7 ± 0.81 |
| | 8 | 5.3 ± 0.47 |
| | 10 | 5.4 ± 0.83 |
| Ascitic fluid drained 8 days after inoculation | 10 | 4.3 ± 0.76 |
| | 12 | 3.4 ± 0.07 |
| | 14 | 6.0 ± 1.06 |
| Ascitic fluid (cell-free) injected 2 days after inoculation | 4 | 3.2 ± 0.25 |

* Each calculation based on 1000 cells, resting and mitotic.
** Standard error of the mean.

interphase cells in mice 2 days after they had been inoculated with 8-day ascitic fluid containing many abnormal interphase cells. Since few such cells have ever been observed undergoing mitosis, it would seem that most of the abnormal cells do not divide again.

The problem arose as to whether the accumulation of ascitic fluid, which coincided with the increase in mitotic abnormalities, was responsible for these abnormalities. The results of draining the ascitic fluid 8 days after inoculation with ascites are shown in Table II. The mitotic picture at 2 days after draining, that is 10 days after inoculation, closely resembled that of 8 days after inoculation, despite the fact that there was only 0.5 to 1.0 cc. of ascitic fluid in each of the drained mice.

TABLE IV.—*Mitotic Index* of the Krebs 2 Ascites Tumor and its Modification by the Ascitic Fluid.*

| | Days after inoculation | Mitotic index |
|---|---|---|
| | 2 | 4.0 ± 0.26** |
| | 4 | 3.6 ± 0.42 |
| | 6 | 3.4 ± 0.78 |
| | 8 | 2.5 ± 0.47 |
| | 10 | 2.4 ± 0.54 |
| Ascitic fluid drained 8 days after inoculation | 10 | 4.1 ± 0.91 |
| | 12 | 3.1 ± 0.81 |
| | 14 | 2.4 ± 0.90 |
| Ascitic fluid (cell-free) injected 2 days after inoculation | 4 | 3.2 ± 0.21 |

* Each calculation based on 1000 cells, resting and mitotic.

** Standard error of the mean.

The main difference was a decrease in the number of abnormal interphase cells in the 2-day-after-draining group. However, in a general way the trend of the after-draining series appeared to be a continuation of that already established in the non-drained group of Table II. The decrease in number of prophases levelled off while the abnormal metaphases continued to increase. The number of abnormal interphases eventually reached the number present before draining. The amount of ascitic fluid increased from about 1 cc. at 2 days after draining to approximately 5 cc. and 10 cc. at 4 and 6 days after draining.

The intraperitoneal injection of 10-day, cell-free ascitic fluid into mice 2 days after they had been inoculated with ascites tumor cells, succeeded in changing the mitotic picture from that found in the controls. When ascites cell samples were taken 2 days after the injection of 4 cc. of ascitic fluid it was found that most of the fluid had been absorbed from the abdominal cavity and only about 0.5 cc.

ascitic fluid was present. The mitotic picture showed a strong inhibition of cells entering prophase and also some increase in abnormal interphase cells.

The mitotic index of the ascites cells in the untreated mice was highest at 2 days, as shown in Table IV, and then decreased until 8 days when it appeared to level off. Draining the ascitic fluid from a mouse 8 days after inoculation resulted in an increase in the mitotic index but then the value fell to that of the 10-day, non-drained ascites mouse. Injecting mice with 10-day, cell-free ascitic fluid did not significantly reduce the mitotic index.

## DISCUSSION AND CONCLUSIONS

The mitotic picture of the ascites cell population in a mouse has been demonstrated to be a changing one and the direction of change is toward an increase in mitotic abnormalities. In metabolic studies of the ascites cells it is important to realize that while only a comparatively few of the total cells are undergoing mitosis, the decrease in the prophases indicates that the interphase cells must be affected.

The ascitic fluid appears to play an important role in the production of mitotic abnormalities. The abnormalities may be brought about either by the decrease in some necessary nutrients due to the competition of an ever-increasing ascites cell population or by the accumulation of metabolic waste products which can then act as mitotic poisons; or it may be due to a combination of these factors. Ascites cells established in a new host have an abundance of nutrient as evidenced by their rapid growth (8). It therefore seems unlikely that an injection of ascitic fluid would cause a deficiency of the nutrient. Changes which occur in the percentages of different mitotic stages are similar to those obtained with mitotic poisons (4). The results of injecting 10-day-old ascitic fluid into recently inoculated mice also seem to favor the view that an accumulation of mitotic poison(s) was the cause of the mitotic abnormalities. Levan (4) describes similar mitotic occurrences in two ascites tumors treated with colchicine: "The typical course of c-mitosis in the present material differs in one striking respect from the c-mitosis as we know it in plants; it is devoid of c-anaphase, the c-metaphase chromosomes enter telophase directly without division of the centromeres and without falling apart of the chromatids." The large number of normal mitoses present in the Krebs 2 ascites tumor, when conditions are favorable for its growth, also supports the view that mitotic abnormalities in neoplasms are due to adverse environments, that is, to necrosis in solid tumors, and to accumulation of waste products in the ascitic fluid of ascites tumors.

## SUMMARY

The number of abnormal mitoses in Krebs 2 ascites tumors has been found to increase progressively. A decrease in the number of cells

entering prophase and also a decrease in the mitotic index has been found to parallel the increase in mitotic abnormalities. The accumulation in the ascitic fluid of substances which act as mitotic poisons appears to play an important role in the production of mitotic abnormalities in Krebs 2 ascites tumor cells.

REFERENCES

(1) T. S. HAUSCHKA, *Tr. New York Acad. Sc.*, Ser. II, 16, 64 (1953).
(2) L. SACHS AND R. GALILY, *J. Nat. Cancer Inst.*, 15, 1267 (1955).
(3) ILSE LASNITZKI, *Brit. J. Cancer*, 7, 238 (1953).
(4) A. LEVAN, *Hereditas*, 40, 1 (1954).
(5) R. J. LUDFORD, *Arch. Exper. Zellforsch.*, 18, 411 (1936).
(6) J. H. TJIO AND A. LEVAN, *An. Aula Dei*, 3, 225 (1954).
(7) A. LEVAN AND T. S. HAUSCHKA, *J. Nat. Cancer Inst.*, 14, 1 (1953).
(8) H. M. PATT AND M. E. BLACKFORD, *Cancer Research*, 14, 391 (1954).

## BOOK REVIEWS

ATOMIC QUEST, by Arthur Holly Compton. 370 pages, 5½ × 8 in. New York, Oxford University Press, 1956. Price, $5.00.

Here is a book that should delight everyone, scientist or layman, who is at all interested in the problems raised in the quest for controlled nuclear fission. It would be difficult to name anyone better qualified than the author to describe the conflicts and the problems that preceded the preparation of the atomic bomb. Dr. Compton's scientific knowledge eminently fitted him for the treatment of the technical aspects, but he is also a man of sensitive and deep feelings who is fully conscious of the spiritual values involved in the release of atomic energy and, in addition, he exercises a shrewd judgment of men and affairs.

Dr. Compton presents his readers with a brief but excellent account of the situation prevailing at the time when hostilities broke out in Europe. He tells how he and a small group of fellow scientists in this country realized the danger that would exist should German physicists succeed in presenting the unrestrainable Hitler with the means of making an atomic bomb. These scientists also realized they were part of their national community and must use their knowledge for the nation's defense and to avoid a world tragedy.

From this moment of realization a race to achieve nuclear fission was opened. Dr. Compton was a major participant in the activities relating to the organization of a tremendous group effort supported by the government. He enlisted the knowledge and talents of the most able physicists, chemists, industrialists, and administrators in order that the vast enterprise might be accomplished. His book is a personal narrative but, as he was in a key position, he saw the whole project in the round. All the principal figures who were engaged in the work flit in and out of the pages of the book, and the author has a deft touch with the pen which enables him to make his men live at the times when action is required of them. Many of the workers who played hardly less important parts are here given credit for the work they performed anonymously. The author is generous with his praise for those who worked for and with him, but he is so reticent about his own share in it that another volume will have to be written before we can fully grasp his influence.

Those people who reproach scientists for having contributed to the means of the wholesale destruction of human beings should find this book of exceptional interest, for they will learn something of the soul-searching that preceded so many agreements to participate in the work. No man took up his share of the burden without fully realizing what the ultimate consequences would be, but each found a moral reason for the undertaking.

Although Dr. Compton writes with restraint the reader is conscious throughout the book of the sense of urgency that surrounded the task. As he gathers together the loose ends from the research laboratories where the atom was being investigated, it is apparent that Germany started from the same mark, but once the government was committed to the search for nuclear fission, the American scientists set a hard pace and dragged the other participants along with them. The book gives a clear conception of the conflicts of opinion that were bound to arise when every forward step was over untravelled ground and where the grimmest hazards hovered about each action. The resultant picture of the achievement of nuclear fission is more complete than anything that has preceded it.

Although the greater portion of the book is concerned with the struggles to make an atomic bomb, the author has much to say upon the other uses of atomic energy and the moral and political implications it may have on world affairs. The book is well documented, so that the technical reader may supplement his knowledge by reference to the original sources.

But, chiefly, the book will stand as a firsthand account of one of man's revolution-

ary accomplishments, written by a modest, sincere, and deeply religious man who held a key position in it. This is a book to be acquired and treasured. T. COULSON

PROCESS CHEMISTRY, PROGRESS IN NUCLEAR ENERGY, SERIES THREE, VOLUME ONE, edited by F. R. Bruce, J. M. Fletcher, H. H. Hyman, and J. J. Katz. 407 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $12.00.

This third volume of the series "Progress in Nuclear Energy" is devoted to a discussion of several chemical processes, which are pertinent to nuclear technology. Since the economic aspects of nuclear power depend on the successful solution of a number of processing problems, this volume is a very welcome addition to the rapidly growing library of the nuclear energy industry. The content of the book is based on a large amount of information, which was released for the first time at the Geneva Conference on Peaceful Uses of Atomic Energy (August 1955).

The book serves admirably to emphasize the chemical aspects of nuclear energy by covering the fields of ore processing, and refining, reactor fuel recovery, and radioisotopes production. The first chapter deals with the chemical aspects of the extraction of uranium and thorium; that is leaching off the ores, concentration of the leach liquor and conversion of the impure concentrate to a pure compound, from which the metal is obtained. Survey papers discuss the different ways of uranium and thorium recovery, so that the reader can have a broad understanding of the techniques used, their relative advantages and disadvantages. On the other end, this chapter includes a paper on the conversion of uranyl nitrate to uranium dioxide and tetrafluoride, giving an appreciation of the difficulties and the details of a particular chemical process. A further paper in this chapter deals with the recovery of thorium from monoazite.

In the second chapter, a general evaluation of separation processes of irradiated fuels is given through a discussion of the typical requirements, the physicochemical principles, practical limitations, and economic aspects of chemical processing in the nuclear energy field. These papers are of a general nature. A discussion on radiation damage of organic reagents used for processing fuels is included.

The third chapter is devoted to the first step in the recovery process of fuel elements: fuel dissolution. Despite the impression that this process may be a simple one, the use of different fuel elements at various stages of nuclear transformation, has demanded a large body of information to control their dissolution. This chapter, also, contains papers of general as well as detailed nature.

The fourth chapter describes the application of solvent extraction to the separation of fission products from uranium and plutonium, the use of tributyl phosphate for the extraction of the nitrates of the actinide elements. A discussion of such variables, as solvent purity, nitric acid concentration, salting agent and temperature is included. The unusual requirements of these extraction processes are clearly brought out in these papers, which are followed by an article discussing the particular problem of the separation of $U^{233}$ from irradiated thorium.

Papers on wet processes for radiochemical separations are grouped in chapter five. For the first time all the chemistry that went into the bismuth phosphate process for the separation of plutonium from uranium and fission products is presented. Other papers of this chapter discuss the use of solvent extraction for reprocessing reactor fuel and blanket materials, for the separation of uranium and plutonium from fission products by means of tributyl phosphate, the processing of uranium-aluminum fuel elements, the separation of $U^{233}$ and thorium from fission products by solvent extraction, some plant experience in the extraction of $U^{233}$, and, finally, liquid-liquid extraction equipment for radioactive processes, and the processing of homogeneous reactor fuel.

Uranium separation by volatilization as uranium hexafluoride is discussed in chapter six. This process represents the most important method to reprocess fuel elements.

Graphs and illustrations are clear; numerous tables are included in the book. Some important nuclear data are collected in the appendix. A glossary of common terms in the nuclear field is included. There is a subject index, but no Author index is given.

The book should prove of great interest to chemists and chemical engineers, and deserves strong recommendation. G. PARRAVANO

INFORMATION THEORY, edited by Colin Cherry. Third London Symposium. 401 pages, diagrams, 6½ × 9¾ in. New York, Academic Press, Inc.; London, Butterworths Scientific Publications; 1956. Price, $11.50.

The London symposiums on Information Theory will remain as classical contributions to the foundation of this new doctrine. The first symposium was held in Sept. 1950 and dealt with the theoretical bases of the subject. The second symposium was mainly devoted to applications to telecommunications.

The present book contains the reports of the third symposium which covered a broad field, from the basic assumptions and definitions down to a variety of practical applications of the theory in optics, telecommunications, taxonomy, linguistics, mechanical translation, semantics, physiology and psychology. The editor, Dr. C. Cherry and his staff certainly did a wonderful job at collecting and getting printed all these valuable papers.

The best way to give an idea of the variety and importance of the subjects discussed in this book, is to give here the table of contents:

Preface
Opening Remarks
Colin Cherry

FUNDAMENTALS

1 Some Consequences of the Finiteness of Information, J. L. van Soest
2 Chance and Control: Some Implications of Randomization, G. Spencer Brown
3 On Some Measures of Information Used in Statistics, M. P. Schutzenberger
4 Optical Transmission, D. Gabor
5 Two-Dimensional Photographic Auto-Correlation of Pictures and Alphabet Letters, W. Meyer-Eppler and G. Darius
6 Ignorance, Knowledge and Information, J. F. Schouten
7 Strategic Information in Games and in Voting, Robin Farquharson

CODING, TAXONOMY, ETC.

8 A General Class of Codes and their Physical Realization, Arthur E. Laemmel
9 Coding for Two Noisy Channels, Peter Elias
10 The Synthesis of Linear Sequential Coding Networks, D. A. Huffman
11 Simple Proofs of Simple Cases of the Coding Theorem, G. A. Barnard
12 A Comparative Study of Delta and Pulse Code Modulation, Lars-Henning Zetterberg
13 Some Clerical Operations and Languages, R. A. Fairthorne
14 Information Retrieval on Structured Content, Calvin N. Mooers
15 On the Language of Taxonomy: an Outline of a "Thermo-statistical" Theory of Systems of Categories with Willis (Natural) Structure, Benoit Mandelbrot

LANGUAGE ANALYSIS AND MECHANICAL TRANSLATION

16 Negative Entropy of Welsh Words, D. A. Bell and Alan S. C. Ross
17 Mathematical Theory of Word Formation, Wilhelm Fucks
18 Suggestions for Mechanical Translation, Silvio Ceccato and Enrico Maretti
19 Influence of Context on Translation, A. D. Booth
20 A Programme for Braille Transcription, J. P. Cleave
21 The Translation of Languages by Machine, Victor H. Yngve
22 Experiments in Mechanical Speech Recognition, D. B. Fry and P. Denes

MEANING AND THE HUMAN SENSES

23 The Place of "Meaning" in the Theory of Information, D. M. MacKay
24 Can Induction be Mechanized?, Vittorio Somenzi
25 New Methods of Speech Transmission, P. Marcou and J. Daguet
26 Effects of Context upon Intelligibility of Heard Speech, D. J. Bruce
27 Auditory Frequency Analysis, J. C. R. Licklider
28 The Cochlear Nucleus and its Relation to Theories of Hearing, J. T. Allanson and I. C. Whitfield
29 An Experimental Treatment of Vision as an Information Source and Noisy Channel, R. L. Gregory

BEHAVIOUR AND ITS MECHANISM

30 Some Properties of a Randomly Connected Neural Network, J. T. Allanson
31 Electrical Simulation of Some Nervous System Functional Activities, W. K. Taylor
32 Factors Limiting the Maximum Impulse Transmitting Ability of an Afferent System of Nerve Fibres, P. D. Wall, J. Y. Lettvin, W. S. McCulloch and W. H. Pitts
33 Pattern Recognition and Learning, Oliver G. Selfridge
34 The Concept of Capacity and the Theory of Behaviour, D. E. Broadbent
35 Studies of Human Channel Capacity, Henry Quastler
36 Application of Communication Theory to the Human Operator, J. D. North

The Following papers are given in summary. They were presented to the Symposium but not read:

37 An Iterative Translation Test, Balth van der Pol
38 A Class of Binary Signalling Alphabets, David Slepian
39 Some Calculations on a Method for Signal Testing, J. W. Cohen and M. M. Jung
40 A Neuronic Model of the Inner Ear, J. Koefoed

It would appear unpractical to give summaries of all 40 papers, but it may be interesting to discuss some of the points of major importance raised in different papers. Dr. van Soest's discussion of the finiteness of information (No. 1) touches a number of very fundamental questions. Information has to be a finite quantity. This means that the physicist cannot measure infinitely small or infinitely large quantities. The continuous structure of the world is a gratuitous assumption and many of the ideas introduced by pure mathematicians are simply useless to the physicist. A similar point of view was developed by this reviewer in a recent book (L. Brillouin, "Science and Information Theory," Acad. Press, New York 1956).

In paper No. 3, M. P. Schutzenberger discusses the possibility of introducing different definitions for "Information." These other definitions can certainly be useful and they will be maintained only inasmuch as

they prove useful, but if we want to avoid confusion, we should coin some other names, and reserve the word "Information" for the quantity defined by Shannon, and now generally recognized. As for the usefulness of new definitions, it is linked with the practicability of experiments leading to the corresponding measurements, and this is a point very often forgotten by a mathematician dealing with statistical methods.

Information obtained from optical experiments is of greatest importance to man. The short papers by D. Gabor and W. Meyer-Eppler touch this fundamental problem. They open a very interesting line of research. It may be well to tell the reader that similar subjects were discussed at another symposium, on Astronomical Optics held at Manchester (April 19–22, 1955) and recently published by the North Holland Publ. Co., Amsterdam. Dr. Gabor also had some excellent remarks in this other symposium.

The papers by J. F. Schouten and R. Farquharson go back to the fundamental definitions, their properties and applications.

The second section groups a number of papers on coding problems, and here we enter the field of communication in a direct way, but these problems also are of great importance for the theory of computing machinery or for a discussion of the logical structure of languages. The reader will find some very interesting discussions and suggestions in the papers of P. Elias, D. A. Huffman, R. A. Fairthorne, C. N. Mooers and B. Mandelbrot. These discussions open the way to the language analysis, discussed in the third section. This reviewer is still rather skeptical on the practical value of mechanical translation, and he thinks that this goal can be attained only after we get a better and more scientific understanding of the logical structure of languages. This is one of the topics discussed in the papers grouped in this section.

The last two sections refer to the practical value of the information for the user. The meaning of information, the question of "intelligibility" in speech transmission with noisy background, the use of information by man and the problem of behaviour.

Every scientist interested in the fundamental or practical problems of "Information" will enjoy reading the book. The papers are excellent and the discussions have been reported in a vivid way. In all congresses, this is always one of the most difficult tasks and the editors of this report must be congratulated for a job very well done.

LEON BRILLOUIN

REACTORS. PROGRESS IN NUCLEAR ENERGY, SERIES TWO, VOLUME ONE, edited by R. A. Charpie, D. J. Hughes, D. J. Littler and M. Trocheris. 492 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $14.00.

Eleven papers are presented which deal for the most part with the characteristics and general properties of certain specific reactors. The geographical distribution of the reactors chosen for discussion is world-wide. In a sense, each paper might be regarded as a historical and technical review of the operation of certain facilities.

The information contained in this book will be more valuable if the reader is already acquainted with the terminology and general theory of reactor operation. In many cases, curves and graphs are presented, which depict such characteristics as power, reactivity, flux, etc., as a function of variables such as, for example, temperature.

Data are also plotted which show the attenuation of neutrons and gamma rays in various shielding materials. However, such curves usually apply to specific reactors and geometries and cannot be applied generally. By the very nature of a historical review, the book is the more useful as a chronicle of experience rather than as any sort of text. The contents would be useful to those contemplating construction of a reactor, because in most instances a day-to-day account of experiences, accidents, etc., is included. For a rapid survey of the characteristics of existing reactors, a "Catalogue of Nuclear Reactors" is presented as the last section of the book.

C. E. MANDEVILLE

AUTOMATIC DIGITAL COMPUTERS, by M. V. Wilkes. 305 pp, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1956. Price, $7.00.

In 1944, a giant new industry was launched with the completion of the world's first automatic digital computing machine at

Harvard University. In view of the tremendous advances made since that time, and the number of different machines developed, it is hardly believable that the automatic digital computer is so young. Industry and business alike are today rapidly changing their old familiar techniques and processes to make room for these new giants which are capable of performing practically any complex mathematical computation in unbelievably short times.

Mr. Wilkes has endeavored to trace the history of the development of these interesting machines and give an introduction to their design in the present text. The result is a rather fascinating account which this reviewer has completely enjoyed. Chapter 1 of the text is devoted entirely to the history of the automatic digital computer starting with the ideas originally proposed by Babbage in the early 1800's and other concepts which led to the development of IBM's Mark I at Harvard University in 1944. Succeeding machines such as the Eniac and Edvac are also briefly described. In Chapter 2, principles of logical design are taken up with the Edvac, a machine designed and constructed at Cambridge, being used as a basis for the discussion.

Succeeding chapters are devoted to such subjects as the principles of program construction, relay computers, storage, electronic switching and computing circuits, and practical considerations in the design and operation of digital computers. An interesting appendix at the end discusses the subject of machinery and intelligence. A bibliography gives a large number of references to pertinent literature on the subject.

Mr. Wilkes has done an excellent job of gathering and presenting the material. The book is, as mentioned before, an introduction and should not be thought of as being a detailed treatment on any of the points included. On the other hand it gives an excellent description of the more basic considerations in the design of digital computing machinery and could be read with profit by all those having an interest in the use of such machines. It would also serve well as an introductory text for students in this particular field.

D. L. BIRX

MOLYBDENUM. Metallurgy of the Rarer Metals No. 5, by L. Northcott. 222 pages, illustrations, 5½ × 8½ in. New York, Academic Press, Inc.; London, Butterworths Scientific Publications; 1956. Price, $6.80.

This book follows the same general format as its predecessors in the series on the rare metals. There is a brief but informative chapter on the history and occurrence of molybdenum followed by a standard treatment on the extraction and refinement of the metal. The chapter on physical properties contains, in addition to the usual properties, a very useful section on preferred orientations. The sections on powder metallurgy, melting, fabrication and mechanical properties are concise, up-to-date and well illustrated. The tables and graphs are particularly well chosen to illustrate the mechanical properties.

Because of its high melting point, systematic studies of the properties of the alloys of molybdenum have been almost nonexistent in the past. The current demand for high-temperature alloys has resulted in great technological advances in the fields of sintering and arc-melting. These advances have made such studies possible. The author has compiled recent results of sintered and melted alloys. Although these results represent only seven different investigations, they are quite detailed.

One of the most valuable contributions that the present series has made is to collect in one volume the equilibrium diagrams for the metal being reviewed. This volume is no exception. It gives information on thirty-two binary systems and several ternary systems. Thirteen binary diagrams are illustrated.

One of the most serious objections to the use of molybdenum is its susceptibility to oxidation at elevated temperatures. This is dealt with at length, and methods of combating the problem are discussed in fairly good detail.

A final section is devoted to the various methods of joining molybdenum. Special emphasis is placed upon the various types of welding. A short 1½ page subject index is included. This text is a welcome addition to the rarer metals series and should prove to be a valuable reference book on molybdenum.

R. L. SMITH

CHEMICAL ENGINEERING KINETICS, by J. M. Smith. 402 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $8.00.

Two approaches had been traditionally used in the treatment of chemical kinetics. The chemist, who is interested in an understanding of the fundamental mechanism of chemical changes, laid down the framework of chemical kinetics in terms of molecules, atoms, ions, and free radicals. The chemical engineer, pressed by production needs, aimed at an empirical framework which could establish the mathematical basis to design and build chemical reactors. For a long time the gap between these two approaches had been wide and open. Recently, however, with the steady development of our knowledge of the field, this gap has been noticeably narrowed down. A contribution in this direction is brought by the book under review. The author has previously published a well known textbook on chemical thermodynamics. He is, therefore, eminently qualified to point out clearly the areas of chemical processes where thermodynamics and kinetics overlap.

After an introductory chapter of a general nature, the Author reviews some fundamental aspects of chemical thermodynamics. Since the book was mainly written at a level suitable for seniors or graduate students, who already should possess the fundamental notions of chemical thermodynamics, it is not clear why this chapter was included. In the third chapter a review of chemical kinetics is given, with particular emphasis on the interpretation of experimental data. This is done by discussing problems taken from recent literature. A treatment of the collision and absolute rate theories is given. The fourth chapter introduces reactor design by discussing first the nature of the problem, the classification of chemical reactors and their operational equations, and secondly some types of reactors. Homogeneous batch reactors are treated in the fifth chapter. The discussion is followed by rather comprehensive examples. In the next chapter the theory of homogeneous flow reactors is discussed. Emphasis is again in deducing rate equations for design purposes from laboratory data. Semi-batch reactors are discussed in the next chapter. A survey of some important aspects of heterogeneous catalysis is carried out in the eighth chapter. The following chapter treats the kinetics of heterogeneous catalytic reactions, including a discussion on mass transfer through the catalytic bed. Heat and mass transfer in reactor is taken up in the tenth chapter.

The design of the gas solid catalytic reactor is presented in the eleventh chapter.

The book is completed by an appendix on the determination of the free energy, enthalpy and entropy of activation by means of the absolute rate theory, and by some additional information on heat transfer through packed beds of solid particles. An author and a subject index are included at the end of the book.

The book will certainly fill the great need for a classroom textbook on the subject, as the previous volume of the same Author on chemical thermodynamics has done.

G. PARRAVANO

ANALOG COMPUTER TECHNIQUES, by Clarence L. Johnson. 264 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.00.

Written from the standpoint of the user, rather than that of the designer, this volume is an excellent introduction for anyone who will be making use of one of the standard general-purpose analog computers to solve engineering problems. The text and the examples given are based on the two most-used computer types, the REAC of the Reeves Instrument Co. and the GEDA, made by Goodyear Aircraft Co. These are typical of the half-dozen or so other makes of general-purpose analog computers which have appeared on the market in the last few years. The author has included, in addition, one chapter on the repetitive analog computer, referring specifically to the G. A. Philbrick equipment; and one chapter on the digital differential analyzer.

As the author emphasizes in his introduction, the engineer, whatever his specialty, will have an increasing need to be familiar with the capabilities and uses of computers, both analog and digital. These have become indispensable tools in engineering and science, as evidenced by the recent acquisition by the Franklin Institute's Laboratories of a UNIVAC I digital computer and a PACE

analog computer, to enlarge its computing facilities and so better serve industry in the Delaware Valley area. BRUCE B. YOUNG

CAMS. DESIGN, DYNAMICS AND ACCURACY, by Harold A. Rothbart. 350 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $9.50.

Designers, instructors and students will find in this book an excellent help in understanding and applying the complex factors involved in successfully developing "a mechanical member for transmitting a desired motion to a follower by direct contact," as a cam is defined.

The author's academic and professional background, and his exhaustive research qualify him to write this thorough, practical and timely book which emphasizes problems involved in high speed machinery, and the mathematic and graphic solutions to same.

Basic curves for displacement, velocity, acceleration and pulse graphically illustrate mathematical equations and their individual characteristics and development are well explained. The interpretation of the results to be expected, the advantages and disadvantages are not only noted, but tabled, compared graphically and commented upon.

In an earlier chapter the author shows how improvements can be effected by combining these basic curves for various portions of a rise or fall, and gives cautions and suggestions. These simple polynomial and trigonometric curves are extended later in the book where polynomials of a higher order are discussed.

Space limitations and the dynamics of the follower aim toward a small cam. This qualification is frequently offset by the pressure angle, size of cam shaft, and sharpness of profile obtainable. A mathematical solution is offered to this problem in lieu of time-consuming trial and error methods. The terms "pressure angle" and "undercutting" are clarified and the effect on size of cam, torque, loads, acceleration, etc., clearly illustrated.

Examples are given and studies made of the effect of various forces (static, inertia, vibratory, frictional spring, torque, etc.) which become critical in high speed mechanics.

Materials of cam, follower and supporting structure are discussed and recommendations made, together with manufacturing technique, experience and machine tool equipment available. All of these considerations enter into the accuracy, precision and durability of the final product.

Mr. Rothbart admits that the actual motion achieved seldom agrees with the theoretical, but he has made a definite contribution in this book. His application of principles, tables to reduce tedious calculations, recommendations, and multiplicity of references, make this book a usable authority on the subject. EARL R. BOWMAN

ENGINEERING MATHEMATICS, by Kenneth S. Miller. 417 pages, diagrams, 6 × 9 in. New York, Rinehart and Company, Inc., 1956. Price, $6.50.

Though written primarily for the first year graduate student, *Engineering Mathematics* is a text equally suited to an advanced undergraduate course in Applied Mathematics. It would appear from the table of contents that the author has not been too generous in the amount of topics offered. However, it would be far more just to accuse him of presenting an extremely well-planned and well-integrated text. He has introduced no topic, to use his own words, "merely for the purpose of exhibiting mathematical gymnastics." Rather, each topic finds justification for its presence elsewhere in the book.

If the level of rigor throughout this book is not to the satisfaction of more sophisticated readers, as intimated by the author, then the level of clarity has become that much higher, especially for those for whom the book was intended. Indeed, the style of writing makes easily digestible the subject matter presented.

Among the topics which are not in themselves novel for a text are Determinants and Matrices, Certain Integrals (Gamma function, Beta functions, Stirling's formula, Euler's Constant, etc.), Linear Differential Equations (Method of Frobenius), Fourier Series and Integrals, and the Laplace Transform. However, some applications are included which are not usually found in texts of this type—for example, correlation functions and the Wiener-Khintchine relations. A generous chapter on Network Theory is also included. But particularly noteworthy is a chapter on Random Functions in which the engineering student is introduced to the theory of Prob-

ability. This chapter includes functions of random, variables, the Normal and other distributions, Stochastic and Ergodic Processes, and some applications in the field of communication engineering. It is this chapter which has been responsible for the inclusion of discussions of Borel sets and Riemann-Stieltjes Integrals in the Appendix.

The references, which are found at the end of each chapter, reflect many recent publications as well as some of the standards such as Bôcher, Ince and Courant. The exercises which are interspersed throughout each chapter were well chosen though not too plentiful.

If this book were to be proclaimed deficient in any sense at all, it would have to be in its disregard of topics such as Complex Variables, Vector Analysis and Numerical Methods, which are generally to be found in texts of this nature. However, the reviewer considers this omission not so much a deficiency as it is a strong argument in support of the author's purpose. Certain things simply did not fit into his plan—more power to him for avoiding the temptation to allude to subjects of mathematical elegance but which would not contribute to the over-all effectiveness of his book.

LAURENCE C. MCGINN

MATHEMATICS FOR ELECTRONICS WITH APPLICATIONS, by Henry M. Nodelman and Frederick W. Smith. 391 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.00.

Rarely does one meet a text which presents such difficulties in assessment as does this one. The authors, who have experience in both teaching and practice, have selected and organized their material from their experience. Presumably they have found the material and form of presentation useful, but it is questionable whether their book will have very general application.

The declared intent of the authors is to emphasize applications rather than mathematics. Their treatment, however, is uneven in level, sometimes being almost "handbook" in nature, and at other times presenting extensive amounts of purely mathematical material. This leaves one wondering what background preparation they expect of the reader for whom they have written.

The book is divided into five parts, each having several chapters. Part 1, "The Uses of Mathematics in Electronics" is not so all inclusive as its title implies. Chapter 1, "Case Histories of Practical Applications" is brief, and includes a few problems which are on the level of the arithmetic puzzles popular in the Sunday supplements. Chapter 2 is devoted to the electronic applications of differential and integral calculus, with numerous examples and problems. The emphasis here is upon relatively routine manipulations, in which the meanings attached to the symbols, whether electronic or otherwise is of no importance. These are just the sort of manipulations the student should have learned thoroughly in his sophomore courses. The more important application of mathematics in formulating the physical problem is dismissed with phrases like "it can easily be shown that. . . ."

Part 2 on "Equation Testing Prior to Mathematical Operations" has chapters on dimensional analysis and its applications. This is an area which is often slighted in formal engineering education, and the treatment is careful and welcome.

The most extensive section of the book is Part 3 on the algebra of circuit analysis, which gives the theory of determinants and matrix algebra with their applications to network solutions, using both mesh and nodal analyses. One might wish that the short section on topology had been placed earlier in the material. In the discussion of determinants the usage of the term "Cofactor" disagrees with the customary definition, but in the section on matrices the usage is correct. Also, on page 160 there occurs the statement that the elements of a determinant always have the same dimensions, but that those of a matrix need not be dimensionally homogeneous. Since the determinant of a square matrix contains exactly the same elements as the matrix, this statement is difficult to understand. Indeed, the solution of Equations 6-49 by Cramer's Rule, although trivial, will lead to determinants whose elements are not dimensionally homogeneous. Except for these troubles, the part is good.

Part 4, "Theory and Applications of Series" has two chapters, entitled "Basic Properties and Applications," which treats primarily power series, and "Non-linear

Electronic Devices," which emphasizes numerical methods. In the first of these the statement of conditions for the comparison tests for convergence or divergence is stronger than necessary. The statement of Theorem 4 contains two misprints. For $|\theta|$ we should have $|u|$, and for $|u|$ we should have $|x|$.

The second of the two chapters includes empirical equations, difference tables, the Gregory-Newton Formula, least squares, harmonic production and intermodulation testing. It is here that we reach the handbook level. The least squares material is essentially recipes, and Fourier Analysis is reduced to the consulting of tables.

Differential Equations are the subject of Part 5. Some space is devoted to the elements which should have been learned in school, and to classical solutions of types of equations which rarely occur in electronics. A brief section on numerical approximations is included. There is no discussion of partial differential equations. On page 263, Equation 11–17 is in error by more than a misprint, and a minor misprint occurs in Equation 11–121.

Transient solutions and Laplace Transforms are covered next, rather well, except that a purist will object to the use of the same functional notation for a time function and its transform. That is, the transform of $f(t)$ is written $f(s)$. Also, the definition of the meaning of the symbol $i(0+)$ as ". . . the initial value of the current $i$ is a very short time after $t = 0$" is poor, although the footnote is correct. Fourier transforms are not treated.

The title of the last part is "The Direction of Mathematics in Electronics." Chapter 13 is a brief exposition of the application of Boolean Algebra to two-terminal switching networks, based on Shannon's work. Unfortunately, the authors have continued the use of the idea of "hindrance," rather than writing in terms of the transmission or "allowance" of the circuit, which is now more generally used.

Chapter 14 is a discussion of the frequency of occurrence of various mathematical techniques in engineering, and lists twelve topics which occur most frequently in electronics. Of these, five are not treated in the present book.

A worthwhile novelty in this text is the mode of obtaining problems for inclusion. A large number have been selected from recent articles in periodicals, and footnote references are appended to the sources. Thus the problems are in large part free of the contrived, artificial, appearance of many other problem lists, and the interested student may easily pursue the applications further.

ALAN C. BYERS

SPECTROSCOPY AT RADIO AND MICROWAVE FREQUENCIES, by D. J. E. Ingram. 332 pages, illustrations, 6 × 9 in. New York, Philosophical Library, Inc., 1956. Price, $15.00.

Ever since the discovery by Fraunhofer (1814) that the spectrum of sunlight is composed of "a multitude of discrete lines," spectroscopists have been improving and extending their methods for measuring all spectral phenomena. Later, by applying Planck's quantum condition, the observed spectral frequencies were used to interpret the energy states of various physical systems. The great influence of this knowledge upon modern physics and chemistry is well known.

More recently the measurement of lower frequencies, that is, in the communications wavelengths, has been undertaken. Most of this work has been in progress only since the development of microwave radar which occurred during the last war. Prior to this time centimeter waves were produced weakly by split-anode magnetrons, the first of these experiments having been carried out in 1933.

Ingram's book is therefore largely devoted to a description of modern high frequency generating and detecting equipment. This part of the subject is covered in considerable detail, sufficient perhaps to allow one experienced in radar and especially waveguide techniques to prepare the necessary apparatus and perform the measurements.

The results and theory of many of these measurements are also taken up in a number of chapters on the determination of chemical and physical parameters. Much of the information thus derived concerns the interactions and binding forces present and is used to test detailed chemical and magnetic theory. A rather interesting running history of the experiments by Rabi, Zacharias, Gardner, Purcell and many others working with radio-

frequency spectroscopy accompanies the theoretical discussion.

Many hundreds of references supplement each chapter, and these are mostly recent (1940's and 50's) owing to the newness of the general subject. It would be necessary to follow the references for detailed theoretical knowledge or experimental data, since the book is rather summary on these topics.

Certainly all students of modern physics will require some familiarity with spectroscopy beyond the optical range, for it is at these low frequencies that the low energy effects are now being observed. These effects are of the utmost importance as we gain a more detailed conception of matter. Ingram's efforts in collecting the theoretical and practical aspects of this important subject have resulted in a convenient and well written little volume of interest to students and professional researchers alike.

C. W. HARGENS

HIGH PRESSURE TECHNOLOGY, by E. W. Comings. 572 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $11.50.

This book contains a discussion on several different topics, which are of importance to high pressure chemical processes. It is a collection of diversified subjects, both fundamental and practical. This approach well gives an over-all view of the high pressure field. However, it is inevitable that a great difference in the level of presentation of the topics presented will result.

The first chapter is devoted to a discussion of the practical need to perform chemical engineering operations at high pressure, while chapter two reviews very briefly, a number of chemical reactions, mostly from the organic field, which are carried out at superatmospheric pressures. In this chapter, ten flow sheets of high pressure processes are included.

In the third chapter an elementary discussion on metals is presented. The emphasis is on mechanical properties, which are most illustrative of metal behavior at high pressures. Chapter four discusses safety factors necessary for successful operation of high pressure equipment. The subject of equipment is discussed in detail in the next chapter. Methods of pressure measurement in practical devices, connections, flanges, fittings, valves, and a review of pump and compressors are included. The theory of thick walled cylinders is the subject of chapter six. The Author discusses, also, means to increase the mechanical resistance of cylindrical containers, like wickel-tape walls, and compound cylinders. Chapter seven is devoted to a presentation of the most important experimental techniques of high pressure research. These include pressure and volume measurements, dew-point, boiling point and vapor-liquid equilibrium apparatus, heat capacity, viscosity of gases, thermal conductivity measurements, autoclaves, and reactors. The thermodynamic behavior of gases and liquids under high pressures is discussed in the eighth chapter, while a brief discussion of the effect of pressure on some individual unit operations is given in chapter nine. The theory of chemical equilibrium, and pressure effects on it are presented in chapter ten.

The eleventh chapter presents a brief discussion of chemical kinetics, diffusion effects in reactors, types of reactors, and reactor theory. The synthesis of ammonia is the subject of the twelfth chapter. The topics treated are the preparation of the raw materials, catalysts, and thermodynamics and kinetics of the reaction, and different processes for ammonia manufacture. No discussion is devoted to other important technological processes, which belong to the high pressure field like synthetic methanol, gasoline and urea. The book terminates with an appendix, on units and dimensions, and a bibliography on experimental work on pVT properties, and a glossary of unusual words.

Some problems with solutions are interspersed into the text of the book. The volume may well serve as a text book for a senior course on advanced topics in chemical engineering and as a reference book for scientists undertaking high pressure research.

G. PARRAVANO

POWER SYSTEM STABILITY, VOLUME III—SYNCHRONOUS MACHINES, by Edward Wilson Kimbark. 322 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $10.00.

Recently much attention has been given to new electronic developments and the large strides which the electronics industry has

made during the past decade. Though receiving less notoriety, the growth and expansion of the power industry during this same period has been equally great. We are reminded by the appearance of Prof. Kimbark's latest volume of Power System Stability that this growth has necessitated the successful solution of the complex problem of meeting the increasing demand while maintaining the stability of the system. The present high degree of reliability of electrical power is a testimony to how well this problem has been solved.

Volume III of "Power System Stability" continues the study begun in Volume I which appeared in 1948, and the latter is considered a prerequisite to the present text. Volume I covers the elements of the stability problem, the principal factors affecting stability, and the ordinary simplified methods of making stability calculations. In the present text, Prof. Kimbark is concerned mainly with the detailed theory of synchronous machines, their excitation systems and a critical examination of the roles played by these machines in the stability calculations. Simplifications which are normally applied in practice are examined carefully, with attempts made to determine the limits within which these simplifications can be safely applied. Methods for dealing with situations in which these simplifications cannot be applied are then derived.

Volume III begins with Chapter 12, devoted entirely to the synchronous machine. The initial section of this chapter defines the various reactances and resistances used in the theoretical development. Other sections discuss suddenly applied short circuits and their effects on the machines both individually and when a part of a larger system. Chapter 13 reviews the various means of exciting these machines and the role played by the exciter in the stability analysis of the network. Damper windings and dampers are considered in Chapter 14. Chapter 15 concludes the text with an extension of the discussion of power system stability which was covered briefly in Volume I.

The text is well compiled, the theory is carefully and logically developed, and numerous examples are provided in each chapter to carry the reader through the practical application of the theory. Diagrams and tables of data taken from actual systems serve to clarify the text. Bibliographies are provided at the end of each chapter to refer the reader to other books and papers adding more detailed discussions of particular phases of the work.

The three volumes now compiled compose one of the most complete treatises available on power system stability.

Dr. Kimbark is well qualified to write on the subject, having specialized in power system problems throughout his career. Volume III should add considerable value to the set as a reference for power system designers and students. D. L. BIRX

CIRCUIT THEORY AND DESIGN, by John L. Stewart. 480 pages, diagrams, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $9.50.

Modern network or circuit theory is founded upon an understanding of the mathematical functions which describe the frequency dependency of the networks, the representation of these functions as complex variables described by their poles and zeros, and the many useful and practical techniques for relating the number and locations of these poles and zeros on a frequency plane to the electrical performance of the networks. These, in general, are the topics covered in this book, with specific consideration being given not only to passive networks, but also to networks which include vacuum tubes, with their equivalent circuits, feed-back circuits, and servomechanisms.

The first three chapters of the book lay the groundwork. A thorough review of steady state network analysis, with emphasis on determinants, for nodal as well as mesh networks, is followed by an introduction to the poles and zeroes which lead to the formulation of network types. The fourth chapter introduces one of the most powerful techniques used in the practical design of circuits by the pole-zero method—that of the normalization of impedances and frequency and frequency transformation shifting. Thus, starting with a network of some basic type, it may be transformed to any one of several types, at different impedance levels and at different center frequencies. Several later chapters expand upon these techniques,

especially when fitting together the several techniques covered in other chapters.

While most of the book is devoted to the pole-zero methods, three chapters are concerned with classical design procedures, those on classical filters and linear vacuum tube equivalent circuits. The final three chapters are concerned with feed-back devices, linear oscillators and functions useful in servo-mechanism theory.

This book is definitely not for use in an undergraduate curriculum unless the student has been adequately prepared with a thorough grounding in calculus, some differential equations and perhaps an introduction to the theory of complex variables. Presumably any graduate student would be fully prepared to attack this book with gusto. Numerous problems are included at the end of each chapter, although many of these are of a general kind. Numerical design problems can be formulated readily by any instructor. The book is easily readable so that any practicing electrical or electronic engineer or circuit designer should have no difficulty grasping the concepts and applying them to original problems.

The author has, in the opinion of the reviewer, brought forth a book which will remain with us for many years as a graduate level textbook and general reference work.

S. CHARP

ELECTROMAGNETICALLY ENRICHED ISOTOPES AND MASS SPECTROMETRY, edited by M. L. Smith. 272 pages, diagrams and illustrations, 5½ × 8½ in. New York, Academic Press, Inc.; London, Butterworths Scientific Publications; 1956. Price, $8.00.

This book is a collection of papers presented at the Conference on Electromagnetically Enriched Isotopes held at Harwell, Berks, England. Delegates from eleven countries attended and the thirty papers which are included in this volume are testimony to the growth of the isotope enrichment program since the war years. These papers were distributed among the seven sessions of the conference.

The electromagnetic separation and enrichment of isotopes is achieved through the application and refinement of well known mass spectrometer techniques. The separated atomic species are then available for subsequent experimentation in a reasonably high state of purity.

The first three sessions of the conference dealt with the construction of ion sources, collector problems and chemical aspects of target preparation. There is an interesting paper in this group which describes the development ion sources which are used for radioactive materials. While this account is brief, the presentation is strongly augmented by clear and detailed illustrations.

Sessions four and five are concerned with the utilization of the isotopes in various fields and the application of the isotopic abundance analysis. Among the informative papers in this group is an account of the abundance technique in determining the age of precambrian rocks. The reader is made aware of the serious problem of error propagation in this type of investigation. This presentation should provide pertinent information for geo-physicists. The wide use of enriched material is emphasized when one reads the section on the distribution of isotopes. For example, the United States Atomic Energy Commission has made over 2600 shipments since 1946. A brief discussion of applications to Solid State Physics is also presented.

The design and operating characteristics of a number of the world's most efficient separators is covered in the sixth session. The papers discuss the installations found in England, on the continent and in the United States. There is some discussion of the problem of ion optics and a theoretical treatment of the effects of magnetic field gradients on beam dispersion.

Additional papers on such nuclear topics as beta and gamma ray spectroscopy, fission and spallation yield studies are found in the seventh chapter.

There are useful bibliographies accompanying many of the papers and a subject index is included. This volume is recommended reading for those working directly with high purity isotopes and those engaged in borderline research activities in which isotopes are used.

J. B. DREW

**Tiny Light Cell to Guide Missiles, Spot Gunfire, or Aid Blind Phone Operators.**—A tiny light-sensitive electronic device that may be used to guide missiles by sunlight, spot the flashes of distant artillery, or enable blind operators to find plug-in positions in a telephone switchboard was described recently by a scientist of the Radio Corporation of America.

Development of the new device, a novel type of photocell no larger than the eraser on the end of a pencil, was disclosed by Dr. J. T. Wallmark, of the technical staff of RCA Laboratories, at a meeting of the Professional Group on Electron Devices, Institute of Radio Engineers. According to details given by Dr. Wallmark, the transistor-like cell is capable of sensing with a high degree of accuracy both the direction and intensity of a source of light. He pointed out that the compact device is capable of performing with improved accuracy and efficiency many functions which have been handled previously only with as many as four separate conventional photocells.

Dr. Wallmark explained that the development is based on the recent discovery of a phenomenon that occurs when light is focused by a lens on a semiconductor junction like those used in transistors to generate and control a flow of electrons. He pointed out that it has long been known that when light shines directly on such a junction, a small voltage is generated across the junction from front to rear. In addition to this previously known effect, according to Dr. Wallmark, it has now been discovered that when the position of the light is changed so that a point of light strikes the junction to one side or the other of its center, a voltage also is developed along the junction from side to side.

"This lateral voltage has been put to use in a new type of photocell whose most interesting characteristic is an additional photo-output that varies from a positive to a negative value over its surface," he said. "The result is achieved by use of three electrical connections instead of the usual two in conventional cells.

"This means that a point of light, focused on the photocell by a lens, will produce a signal between two of the connections that varies with the angle between the direction to the light and the axis of the cell. These signals can be read with high accuracy to determine the direction and intensity of the light source. At the same time, the third electrical connection produces a normal photocell signal in response to the intensity of the light and independent of its direction."

Dr. Wallmark discussed these possible applications for the new device:

*Light communication between moving objects:* The photocell could be used to receive light signals from a transmitter. As explained by Dr. Wallmark, "the transmitted message is picked up by the photocell using the normal connection, while the new direction-sensitive connections may be used for lining up or 'tracking' receivers and transmitters continuously even if one or both stations are moving."

*Guided missile navigation:* Since the

photocell is capable of detecting the center point of the sun "with great accuracy," according to Dr. Wallmark, it could form the heart of an automatic navigation system employing the sun as a guide.

*Artillery spotting and range finding:* Two spaced photocells would be used in this application. Dr. Wallmark explained that the cells could detect a sudden flash of light at a distance, such as that caused by the firing of a gun, and could determine the direction to the light. The intersection of these directional lines extending from each cell would then locate accurately the position and the distance of the gun. Dr. Wallmark also pointed out that the photocell would be capable of locating sources of infra-red or ultra-violet light as well as visible light.

*Light finder for blind telephone operators:* Dr. Wallmark stated that the extreme compactness of the new photocell would make possible the design of a lens and photocell assembly small enough to mount on a finger in the manner of a ring. The cell would signal the direction to an indicator lamp locating a call on the switchboard. As explained by Dr. Wallmark, the signal from the photocell would be fed through a special circuit that would translate it into a low audible frequency for transmission into an earphone. This audible signal would fade as the operator's hand neared the right location on the switchboard.

In addition to these applications, Dr. Wallmark pointed out that the new cell might be used also in a variety of detecting and measuring functions in which signals provided by a light source are translated into various mechanical effects. As an example, he mentioned the production of sound from a film sound track.

**Shock Tube Overcomes Power Loss.**—Power loss from piston knock may be largely overcome in the family auto through scientific study being carried on by means of a new device called a "shock tube." Experts at Ford Motor Company's Scientific Laboratory made the shock tube as a device for producing fuel-air explosions that can be photographed. The shock tube is being used to study just what causes different fuels to give good or poor performance.

Power-robbing piston knock is caused by pockets of fuel exploding in an uncontrolled manner in the combustion chamber of a car. In future engines, the problem may be to get fuels to ignite at just the right instant and to burn at exactly the right rate to deliver the most power. In a car's engine, fuel is fired by a spark plug, but the Ford scientists needed a more uniform means of touching off the explosion. They used supersonic shock waves created within a 25-ft.-long rectangular tube to do it.

The Ford men produce the shock waves in the $2 \times 3$-in. tube by releasing compressed helium gas. The waves rush from one end of the tube two or three times the speed of sound. When they strike a fuel-air mixture in the other end of the tube, a controlled explosion occurs. Photographs, taken in 15 one-millionths of a second, show the scientists what happens to the molecular structure of fuel and air when the explosions occur.

When the Ford scientists learn what causes uncontrolled explosions, they will be well along the road to discovering how to prevent them and make the family auto more efficient.

**Atomic Hydrogen Unit.**—A complete redesign of its atomic hydrogen welder has been announced by the General Electric Company's Welding Department.

The atomic hydrogen process is particularly useful in the repair of dies, molds, machine tools, and oil well bits, and with materials such as high alloy steels, chrome, nickel, and molybdenum. In the atomic hydrogen process, a jet of hydrogen is directed across the arc formed between a pair of tungsten electrodes. This generates atomic hydrogen which recombines at a short distance from the arc to form molecular hydrogen. This action, in turn, results in intense heat concentration in a small area shielded by a hydrogen atmosphere. The operator can control the heat applied to the work by either varying the distance separating the two tungsten electrodes or by varying the distance of the arc from the work. Metal of approximately the same analysis and strength as that being welded can be deposited, usually without the use of flux, according to General Electric engineers.

Standard equipment on the atomic hydrogen unit is an automatic hot start device that gives a current surge for positive starting, particularly at low current settings.

Standard equipment are power factor capacitors, which help reduce line current requirements and power costs, according to the engineers.

Gas flow in the process is controlled automatically, flowing only when the arc is struck between the two electrodes. Gas consumption ranges from 30 to 90 cu/ft/hr during manual operation.

Current range of the unit is 7–100 amp.

**Giant Brain to Keep Medical, Hospital Records.**—A new electronic "brain" that will keep track of the hospital and medical records of more than 3½ million Michigan residents has been ordered by the Michigan Hospital Service. Robert J. Koch, director of the Office Management Division of MHS, told delegates to the 1956 Blue Cross–Blue Shield Management Workshop session in Chicago that the new computer system will through electronic means "considerably reduce the problem of handling the mountains of data that have to be processed daily."

Developed by Datamatic Corporation, the new "brain" is a large-scale general-purpose electronic data-processing system known as Datamatic 1000. It will occupy 5000 sq. ft. of air conditioned, humidity-controlled space and will consist of 12 sections. These include a central "brain" which can "read" and "write" at the rate of 60,000 digits per second, simultaneously handling 1000 multiplications, or 4000 additions or 5000 comparisons. The "brain" is scheduled for delivery next July.

Daily transactions—name and address changes, changes in coverage and so on—are fed into the processing system's input converter (a special transcription device) which translates, edits and transcribes the data onto 3-in. wide magnetic tapes. One reel of tape (2700 ft. long) can store 37,200,000 digits of information—or the equivalent of information contained on 465,000 punched cards.

Koch explained that the "brain" to be used by Michigan Hospital Service and Michigan Medical Service will incorporate eight magnetic file units to memorize and store the tens of thousands of individual statistics now maintained on punched cards and file index cards.

Michigan Hospital Service and Michigan Medical Service, the second largest Blue Cross–Blue Shield service in the country with 1,400,000 subscribers (covering some 3½ million

people) conducts a minimum of 25,000 transactions or record changes daily. These range from admission requests, questions relative to subscriber coverage, to name and address revisions, service reports and many others. The Datamatic system will be able to handle these 25,000 revisions within two hours, leaving an additional six hours for such operations as billing procedures, compilation of over-all MHS and MMS statistics necessary for efficient operation.

A special auxilliary unit incorporated into the Datamatic 1000 system, called a File Reference Unit, makes it possible to refer to or question any one of the eight magnetic file units without disturbing the rest of the system.

Any subscriber's or member's status or any person's complete medical and hospital history or any part of it, can be located and reported on in less than two minutes.

The giant electronic brain, which MHS-MMS will lease from Datamatic will not only make it possible to reduce costs in carrying out routine office procedures, but will offer the critical advantage of continuous compilation, updating and processing of subscriber records upon which daily decisions can be made, Koch said.

The "1000" system, Koch said, "talks back" in any one of three ways. For data needed during the day, the information is released either in punched card form (100 cards per minute) or in printed form (900 lines per minute). If the data are to be used by the system itself (for internal tallying purposes), it comes out on magnetic tape in codes which only the machine itself can read.

Because of the extensive built-in self-checking features in the system the only possibility of an error rests with the people feeding it data. To minimize this contingency, MHS and Datamatic have been training supervisors and methods and procedures personnel who will oversee the care and feeding of the giant brain. It generally takes about two years advance preparation to make the change over to electronic data processing systems. Michigan Hospital Service people have about 13 months of this preparation already behind them.

The Datamatic System, newest of the giant "brains" now on the market, is the result of the pooling of talents of Minneapolis-Honeywell Regulator Company and Raytheon Manufacturing Corporation, parents of Datamatic Corporation.

**Joseph Priestley Home Being Restored.**—The home of Joseph Priestley has been acquired and is being restored to its original appearance by the Borough of Northumberland. This house, started in 1795 and completed in 1797, and owned for many years by the Priestley family, was purchased in 1920 by The Pennsylvania State University and presented to Northumberland in 1956. The home is of Georgian architecture, is in fine condition, and occupies an acre of landscaped ground near the Susquehanna River. Under the direction of the Priestley Memorial Association, the Widow's Walk, kitchen and laboratory are being restored to their original appearance and the rooms refurnished with American furniture in style around 1800.

The Association desires to gather all obtainable information on Priestley and on his American home. It would like to receive reprints of articles bearing on his work, and its welcomes correspondence with anyone interested in this matter. Visitors are welcome. Correspondents may ad-

dress Mr. Lewis K. Rich, 464 Front St., Northumberland, Pennsylvania.

The Division of History of Chemistry of the American Chemical Society is co-operating with the Priestley Memorial Association in the restoration of Priestley's laboratory. To this end a committee including Sidney Edelstein, Claude K. Deischer and Wyndham Miles, chairman, has been constituted. The committee wishes to locate apparatus of Priestley's American period, with a view of acquiring them or having replicas made. It welcomes correspondence with anyone knowing the whereabouts of apparatus, and with scholars interested in this project. Correspondents may address Dr. Wyndham Miles, Edgewood, Maryland.

**Army Engineers Assisting in Developing Satellite.**—The Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Virginia, is assisting in the development of the first man-made satellite for "Project Vanguard." Dr. George Hass, chief of the Physics Research Laboratory, has been assigned the task of both developing and applying a coating for the sphere to be sent into outer space.

Purpose of the coating is three-fold:

1. Give high reflectance to light to minimize heating by absorption of sun radiation.

2. Act as a highly absorbing surface in the infrared to stabilize temperature by high radiative power in the infrared.

3. Prevent corrosion of the magnesium surface.

Previous experiments in coating aluminum spheres with silicon monoxide have proved successful, but since the man-made satellite will be of magnesium it will be necessary to develop a highly oxidized silicon monoxide that will adhere strongly even during large temperature changes and to protect it from corrosion.

It is proposed to first coat the magnesium with chromium, then with aluminum for high visible reflectance, and finally with silicon monoxide, which is expected to form a more strongly adhering coating than would silicon monoxide directly on magnesium. It will also protect it from oxidation. Magnesium samples have been coated with the proposed film combination and have shown the desired properties.

The satellite, which is expected to help scientists learn more about the earth's shape and conditions in outer space, will be a near-perfect sphere. Although it will be of magnesium, because of its lightness, it will have a sturdy inner structure to support instruments.

Four radio antennae will project outside the sphere to transmit information on the earth's shape, density of the outer atmosphere, temperatures inside and outside the ball, and other pertinent information which cannot be recorded inside the shell because the ball is expected to burn up from heat caused by friction when it falls back to earth.

The sphere will be propelled into outer space by a three-stage rocket. It is to be launched at Patrick Air Force Base, Cocoa, Fla. The data of launching has not yet been revealed. It is believed that the sphere will follow an elliptical path that will take it to within 200 miles of the earth and, perhaps, as far away as 1500 miles. Its brilliance is expected to make it visible in some areas of the country.

The Naval Research Laboratory has over-all responsibility for the project in which many firms and agencies are participating, and which is sponsored by the National Academy of

Sciences for the International Geophysical Year starting next July.

**Magnetic Plate Handling Attachment for Lift Trucks.**—The Yale & Towne Manufacturing Company of Philadelphia has announced the development of a magnetic attachment for industrial lift trucks to insure fast, safe handling of metal plate. The attachment, mounted between the forks of a Yale electric truck, operates automatically when a control switch on the cowl of the truck is placed in the "on" position. Turning the switch "off" permits regular fork handling with the lift truck. More efficient handling of metal plate is possible with this new Yale development since the action of the magnet prevents the load from slipping or tipping.

The magnet between the forks becomes energized (the switch being on) as soon as the load depresses a bar switch located in the middle of the magnetic plate. This action also lights a small light on the cowl showing the operator his magnet is in operation.

As a safety precaution, the forks on the truck are pivoted at their base. A tension spring at the top of the fork holds them in a straight upright position when empty, while the weight of a properly placed load moves the fork backs forward slightly against a pair of stops. If a load is not placed properly on the forks, one of them will not move forward. When this happens, the hoist mechanism of the truck will operate, but the truck cannot be moved forward or reverse. Upon placement of the load, the magnet is de-energized automatically by the action of the fork backs which are pulled to the rear by their springs as soon as contact with the load is broken.

There is a Demag button centered with the On-Off switch and the signal light on the truck cowl. This is for emergency use to over-ride the action of the magnetic attachment in case of unusual circumstances where an operator might be stuck in an inoperable situation with an improperly placed, partially raised load.

**Methyl Chloroform as Safe Solvent.**—The findings reported recently by the Hygienic Guides Committee of the American Industrial Hygiene Association indicate the remarkable safety and use of an all but unknown solvent material, methyl chloroform. This committee of scientists, specialists in the control of industrial health conditions, report their findings after review of the thorough animal experimentation tests conducted on the material.

The solvent which has become commercially available only recently, looms as a substitute for the dangerous carbon tetrachloride. Although almost identical in physical and cleaning properties it is shown to be much less likely to injure the user in almost every application where hand cleaning at room temperature is done.

In the case of methyl chloroform, general ventilation is considered sufficient for brief exposures or those resulting from very small quantities of the solvent. With prolonged exposures or with elevated temperature operations, process ventilation will be required. Recommended control procedures for high hazard carbon tetrachloride state, "If carbon tetrachloride must be used, the operation should be adequately ventilated and isolated from the general working space."

Recommended Maximum Atmospheric Concentration (8 Hours) for methyl chloroform is 500 parts of

vapor per million parts of air, by volume (ppm), while that for carbon tetrachloride is 25 ppm.

A comparison of the two solvents shows their great similarity for cleaning purposes. Both are insoluble in water, but are miscible with alcohol, ether, acetone, benzene, and with each other. Methyl chloroform will react violently with *unoxidized* aluminum and certain other metals, but it is now readily available from five or six commercial sources in stabilized form to eliminate this undesirable property of the technical grade chemical.

Concise control standards for human exposure to Ethyl Alcohol, Hydrogen Fluoride, Mercury, Trichloroethylene and 1,1,1,-Trichloroethane (Methyl Chloroform) are given in this recent issue of the Association's "Hygienic Guide Series." Hygienic Guide sheets on each substance may be obtained from The American Industrial Hygiene Association, Inc. 14125 Provost, Detroit 27, Michigan, at 25 cents each, prepaid.

**Special Color TV Camera for Closed-Circuit or on-Air Surgical Colorcasts.** —The Radio Corporation of America has started commercial production of the first compatible color television camera system specifically designed for medical use.

Engineered for completely remote operation and maximum operating economy, the RCA medical TV camera is expected to be widely used in hospitals and medical centers, colleges and universities, and research institutions for origination of live closed-circuit or on-air colorcasts of surgical, medical, and educational instruction and demonstrations, according to E. C. Tracy, Manager, RCA Broadcast and Television Equipment Department.

Deliveries of the medical TV camera (TK-45) were begun in December, the first units going to Walter Reed Army Medical Center, Washington, D. C.; Smith, Kline and French Laboratories, Philadelphia, Pa., pharmaceutical firm; and the University of Michigan Medical School, Ann Arbor, Michigan. Other orders are being accepted for Spring delivery.

"The RCA medical TV camera has been recognized as an electronic development of major significance to medical research and education," said Mr. Tracy. "It makes available to the medical and educational instructor an important new classroom aid which combines the immediacy of television, the realism and detail of compatible color, and the quality performance of broadcast equipment with its own specially developed features for maximum operating flexibility and economy."

The medical camera is designed around three vidicon camera tubes and special electronic circuitry which make possible televising surgical procedures in full-color detail under normal operating-room lighting. It measures only 26 by 15 by 14 inches, weighs less than 200 pounds and, to eliminate interference with surgery or demonstrations, is designed for permanent installation in an overhead fixture which supports both camera and surgical lamp.

Mr. Tracy said that the new design approach in the medical camera provides users with numerous operating and performance advantages. It permits long periods of exposure to a single scene without danger of image burn-in, and virtually eliminates "halo" or overloading effects caused by reflections from polished instruments or wet tissue.

New operating flexibility and detail are provided by the camera's "overhead" design, which keeps the camera

out of interference range of the surgeon or instructor, yet permits unrestricted viewing freedom. Horizontally mounted, the camera peers into a mirror which reflects the optical path downward through an opening in the surgical lamp. Remotely controlled, the mirror can be panned and tilted to change the viewing field without moving camera or lamp. If the lamp is moved, the camera's optical path automatically moves to coincide with the change in lighting path.

The camera's complete remote-control design permits one-man operation of the entire camera system, including camera focusing and selection of lenses, as well as operation of the camera mirror. A lens turret accommodates four different lenses, for long, medium, short, and close-up viewing. In cases of surgery confined to a tiny area, such as the eye, optical magnification of the televised area can be obtained by selection of lenses with long optical focus.

The TK-45 camera also can be incorporated into a microscope system for direct televising of microscopic specimens, enabling a single instructor to display full-color microscope enlargements to an entire class or to groups of classes tied into a closed-circuit network. The camera can be used with virtually all general purpose laboratory microscopes.

The fully compatible color television camera system, in addition to closed-circuit applications, can be used to feed programs to local or television network facilities for broadcast transmission.

Mr. Tracy pointed out that the system's remote control room can be located as far as 500 feet from the camera. All necessary studio control equipment is housed in a master control console.